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MATHÉMATIQUES

DÉTERMINATION D'UNE SÉRIE DE FONCTIONS DONNÉES D'APRÈS LES VALEURS DES EXTREMA DE SES RESTES SUCCESSIFS

Par SERGE BERNSTEIN, de l'Académie

Soit $\varphi_n(x)$ une suite de fonctions continues définies sur un ensemble fermé E de points x, telle que $\varphi_n(x) \leqslant \varphi_n(a) = 1$, où a est un point donné de l'ensemble E, satisfaisant de plus à la condition

$$\varphi_n(x) \geqslant \varphi_{n+1}(x) \geqslant 0, \tag{1}$$

quel que soit $x \subset E$.

Considérons la classe des fonctions $f(x)$ représentables par des séries

$$f(x) = \sum_1^\infty b_n \varphi_n(x), \tag{2}$$

convergentes pour $x = a$; d'après le théorème d'Abel, ces séries seront par conséquent uniformément convergentes en tous les points $x \subset E$.

Désignons par $\varepsilon_n N_n$ l'extremum absolu du reste

$$R_n(x) = \sum_{n+1}^\infty b_k \varphi_k(x),$$

où $N_n \geqslant 0$, $\varepsilon_n = \pm 1$.

Théorème I. *Une fonction $f(x)$ de la classe (2) est entièrement déterminée par les valeurs $\varepsilon_n N_n$ des extrema absolus de tous ses restes ($n \geqslant 0$) sur l'ensemble E.*

En effet, formons la différence

$$F(x) = \sum_1^\infty a_n \varphi_n(x)$$

entre deux fonctions dont les restes successifs auraient les mêmes extrema $\varepsilon_n N_n$. Il existerait alors ou bien deux points ξ_n et ξ_n appartenant à E, où

$$\rho_n(\xi_n) = \sum_{k=n+1}^\infty a_k \varphi_k(\xi_n) > 0, \quad \rho_n(\xi'_n) < 0,$$

ou bien au moins un point x_n, où $\rho_n(x_n)=0$ avec $\varphi_{n+1}(x_n)>0$. Or, le fait que $\lim_{n=\infty} \rho_n(a)=0$ entraîne [si $F(x)$ n'est pas identiquement nul] qu'il existe une valeur de n telle que

$$|\rho_n(a)|>|\rho_{n+h}(a)|$$

pour toute valeur de $h>0$. Par conséquent, en remarquant que

$$\rho_n(x)=\rho_n(a)\,\varphi_{n+1}(x)+\rho_{n+1}(a)\,[\varphi_{n+2}(x)-\varphi_{n+1}(x)]+ \\ +\rho_{n+2}(a)\,[\varphi_{n+3}(x)-\varphi_{n+2}(x)]+\dots \qquad (3)$$

[à cause de $a_{n+1}=\rho_n(a)-\rho_{n+1}(a)$], nous obtenons, en tenant compte de (1), que pour cette valeur de n

$$|\rho_n(x)-\rho_n(a)\,\varphi_{n+1}(x)|<|\rho_n(a)|\,\varphi_{n+1}(x)$$

en tous les points $x\subset E$, où $\varphi_{n+1}(x)>0$. On aurait donc nécessairement

$$\rho_n(x)\cdot\rho_n(a)>0,$$

ce qui exclut l'existence des points ξ_n et ξ'_n, où $\rho_n(\xi_n)\cdot\rho_n(\xi'_n)<0$, aussi bien que l'existence des points x_n, où $\rho_n(x_n)=0$ [avec $\varphi_{n+1}(x_n)>0$]. Ce théorème d'unicité doit être complété par un théorème d'existence analogue à celui qui concerne les meilleures approximations d'une fonction que j'ai donné à un autre endroit *.

Théorème II. *Quelle que soit la suite infinie de nombres donnés*

$$N_0\geqslant N_1\geqslant\dots\geqslant N_n\geqslant N_{n+1}\geqslant\dots\geqslant 0$$

tendant vers zéro, il existe une fonction (unique)

$$f(x)=\sum_1^\infty b_n\varphi_n(x),$$

telle que les extrema de ses restes successifs $R_n(x)=\sum_{n+1}^\infty b_k\varphi_k(x)$ *sont égaux à* $\varepsilon_n N_n$, *où* $\varepsilon_n=\pm 1$ *sont fixés à volonté* (il résultera de la démonstration qu'aucun des restes $R_n(x)$ ne peut atteindre son extremum avec les deux signes).

En effet, observons (comme à l'endroit cité) que, si

$$f_1(x)=\sum_1^\infty b'_n\varphi_n(x),\quad f_2(x)=\sum_1^\infty b''_n\varphi_n(x)$$

et

$$F_n(\lambda)=\text{maximum}\left|\sum_{n+1}^\infty(b'_k+\lambda b''_k)\,\varphi_k(x)\right|,$$

la fonction $F_n(\lambda)$ est une fonction convexe du paramètre λ. Supposons, en particulier, que $f_2(x)=\varphi_{n+1}(x)$, $b'_k=0$ pour $k\leqslant n+1$ et

$$F_{n+1}(0)\leqslant N_n.$$

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* Comptes Rendus, t. 206, 1938, «Sur le problème inverse de la théorie de la meilleure approximation».

Dans ces conditions, $F_n(0) = F_{n+1}(0)$ et, par conséquent, l'équation

$$F_n(\lambda) = \text{maximum} \left| \lambda \varphi_{n+1}(x) + \sum_{n+2}^{\infty} b'_k \varphi_k(x) \right| = A \qquad (4)$$

possèdera pour toute valeur de $A > F_{n+1}(0)$ une solution unique positive $\lambda_{n+1} > 0$ et une solution unique négative $\lambda'_{n+1} < 0$; d'ailleurs, à cause de $\varphi_{n+1}(x) \geqslant 0$, l'extremum de

$$\lambda_{n+1} \varphi_{n+1}(x) + \sum_{n+2}^{\infty} b'_k \varphi_k(x)$$

sera positif et celui de

$$\lambda'_{n+1} \varphi_{n+1}(x) + \sum_{n+2}^{\infty} b'_k \varphi_k(x)$$

sera négatif. Dans le cas, où $A = F_{n+1}(0)$, l'équation (4) admettra, sauf la solution évidente $\lambda = 0$, encore une solution unique de signe opposé à celui de l'extremum de

$$\sum_{n+2}^{\infty} b'_k \varphi_k(x),$$

lorsque ce dernier n'est atteint qu'avec un seul signe.

Ainsi, il y aura toujours une valeur parfaitement déterminée $c_{n+1} \geqslant 0$ telle que

$$\text{extremum} \left(\varepsilon_n c_{n+1} \varphi_{n+1}(x) + \sum_{n+2}^{\infty} b'_k \varphi_k(x) \right) = \varepsilon_n N_n,$$

où $c_{n+1} > 0$, si $N_n > F_{n+1}(0)$.

En répétant le même raisonnement, nous pouvons former ainsi une fonction

$$f^{(n)}(x) = \sum_{i=0}^{n} \varepsilon_i c_{i+1} \varphi_{i+1}(x) + \sum_{n+2}^{\infty} b'_k \varphi_k(x)$$

telle que ses restes successifs $R_i(x)$, pour $i \leqslant n$, admettent, respectivement, des extrema égaux à $\varepsilon_i N_i$ [d'après ce qui précède, on a $c_{i+1} = 0$ dans le cas seulement, où $\varepsilon_i N_i = \varepsilon_{i+1} N_{i+1}$, et l'extremum de $R_i(x)$ n'est atteint qu'avec un seul signe, lorsque $c_{i+1} > 0$; donc, si la fonction initiale $f_1(x)$ a été choisie de façon $F_{n+1}(0) > N_n$, tous les restes $R_i(x)$ $(i \leqslant n)$ atteignent leurs extrema avec un seul signe].

Soit, en particulier, $f_1(x) = 0$; alors

$$f^{(n)}(x) = \sum_{i=0}^{n} \varepsilon_i c_{i+1} \varphi_{i+1}(x)$$

sera un polynôme, et il nous reste à montrer que $f^{(n)}(x)$ tend uniformément vers une fonction déterminée $f(x)$, lorsque $n \to \infty$. En effet, posons

$$f^{(n+k)}(x) - f^{(n)}(x) = \sum_{1}^{n+k+1} a_i \varphi_i(x), \quad (k > 0)$$

$$\rho_m(x) = \sum_{m+1}^{n+k+1} a_i \varphi_i(x), \quad (m \geqslant 0).$$

On a, par définition, $|\rho_m(x)| \leqslant N_m \leqslant N_n$ pour $m > n$. Donc, en vertu du raisonnement qui a servi à démontrer le théorème I, on doit avoir

$$|\rho_m(a)| \leqslant N_n,$$

quel que soit $m \leqslant n$, et à cause de (3)

$$|\rho_m(x)| \leqslant 2N_n$$

en tous les points $x \subset E$. Ainsi, en particulier, pour $m = 0$, on a

$$|f^{(n+h)}(x) - f^{(n)}(x)| \leqslant 2N_n,$$

d'où il résulte que $f^{(n)}(x)$ tend uniformément vers une fonction $f(x)$ de la forme (2) qui satisfait aux conditions du théorème.

Indiquons encore la proposition suivante:

Théorème III. *Si les fonctions* $\varphi_n(x)$ *satisfont à la condition*

$$\varphi_n(x) - \varphi_{n+1}(x) \geqslant \varphi_{n+1}(x) - \varphi_{n+2}(x) \geqslant \ldots \geqslant 0 \tag{5}$$

[*plus restrictive que la condition* (1)], *la fonction* (2) *déterminée par les conditions du théorème* II, *où* $\varepsilon_i = (-1)^{i+1}$, *est égale à*

$$f(x) = \sum_0^\infty (-1)^{n+1} [N_n + N_{n+1}] \varphi_{n+1}(x).$$

En effet, tous les restes $R_n(x)$ prennent au point a la valeur $(-1)^{n+1} N_n$, et on a pour tout point $x \subset E$

$$N_n \varphi_{n+1}(x) \leqslant (-1)^{n+1} R_n(x) = N_n \varphi_{n+1}(x) + N_{n+1}\big(\varphi_{n+1}(x) - \varphi_{n+2}(x)\big) -$$
$$- N_{n+2}\big(\varphi_{n+2}(x) - \varphi_{n+3}(x)\big) + \ldots \leqslant N_n \varphi_n(x).$$

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MATHEMATICS

IDEALS AND SUBRINGS OF THE RING OF CONTINUOUS FUNCTIONS*

By G. ŠILOV

(Communicated by I. M. Vinogradow, Member of the Academy, 17; XI; 1938)

If the set of elements of an algebraical ring forms at the same time a topological space, in which the ring operations of summation, subtraction and multiplication are continuous in both arguments, we say that we have before us a topological ring.

In considering the ideals of topological rings, it is natural that we should confine ourselves to ideals which, from the topological point of view, are closed sets. In what follows we shall understand under ideals of topological rings only such closed ideals.

Every element α of a topological ring generates a minimum ideal $I(\alpha)$ containing it. The ideal $I(\alpha)$ may be defined as the intersection of all ideals containing α, or as the closure of the set of all products of the type $\beta\alpha$, where β is an arbitrary element of the ring. Every ideal generated by some element of the ring is called chief ideal.

Consider the ideals of the ring $C(S)$ of all complex continous functions $\alpha(x)$, defined on a certain compact metrisable space S. We shall suppose that the topology in the ring $C(S)$ is defined in the sense of uniform convergence.

Theorem I. *The chief ideal $I(\alpha)$ generated in the ring $C(S)$ by the function $\alpha(x)$ coincides with the set of all functions from $C(S)$ vanishing on the set N_α, on which $\alpha(x)=0$.*

Proof. It is obvious that every function from $I(\alpha)$, being the limit function of functions of the type $\beta\alpha$, vanishes at all points of the set N_α. We have to prove that, conversely, every function $\beta(x)$ vanishing at all points of N_α belongs to $I(\alpha)$. To this end consider the sequence of functions $\beta_n(x)$ defined as follows:

$$\beta_n(x)=\beta(x), \text{ if } \rho(x, N_\alpha)\geqslant\frac{2}{n},$$

$$\beta_n(x)=n\left\{\rho(x, N_\alpha)-\frac{1}{n}\right\}\beta(x), \text{ if } \frac{1}{n}\leqslant\rho(x, N_\alpha)\leqslant\frac{2}{n},$$

$$\beta_n(x)=0, \text{ if } \rho(x, N_\alpha)\leqslant\frac{1}{n}.$$

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* After relieving this paper to press, the author has noted that theorem I and theorems, essentially equivalent to theorems III—IV, have been devised in a more complicated way in a recently published paper by M. Stone (2).

It is easily seen that if $\beta(x)$ vanishes at all points of the set N_α, the sequence $\beta_n(x)$ converges uniformly to $\beta(x)$. Put now

$$\gamma_n(x) = \beta_n(x) : \alpha(x), \text{ if } \rho(x, N_\alpha) \geqslant \frac{1}{n},$$

$$\gamma_n(x) = 0, \text{ if } \rho(x, N_\alpha) \leqslant \frac{1}{n}.$$

Clearly, the functions $\gamma_n(x)$ are continuous on the whole space S. Since, moreover,

$$\beta_n(x) = \gamma_n(x)\,\alpha(x),$$

all functions $\beta_n(x)$, and consequently also the function $\beta(x)$, belong to the ideal $I(\alpha)$, q. e. d.

Theorem II. *All ideals of the ring $C(S)$ are chief ideals.*

Proof. Consider any ideal I of the ring $C(S)$. Denote by N_I the set of all points of the space S, at which all functions of the ideal I vanish. Let

$$\alpha_1(x),\ \alpha_2(x),\ \ldots,\ \alpha_n(x),\ \ldots$$

be a sequence of functions belonging to the ideal I and everywhere dense on it. Obviously, for every point x of the set $S - N_I$ there can be found a function of this sequence, which is different from zero at the point x. The function

$$\alpha(x) = \sum_n a_n\,\alpha_n(x)\,\overline{\alpha_n(x)},$$

where the constants a_n are positive and chosen in such a way that the series on the right hand side converges uniformly, is positive at every point of $S - N_I$ and belongs to the ideal I. Consequently, the ideal $I(\alpha)$ is contained in the ideal I. But since $I(\alpha)$ contains all functions vanishing at every point of N_I (see Theorem I), we conclude that, conversely, I is contained in $I(\alpha)$. Thus, we have proved that the ideal I coincides with the chief ideal $I(\alpha)$.

Theorems I and II show that there exists a one-to-one correspondence between the ideals of the ring $C(S)$ and the closed sets of the space S: to every ideal I corresponds the closed set N_I and I itself is nothing else than the set of all functions from $C(S)$ vanishing at every point of the set N_I.

Further, in order that the ideal I should contain the ideal I', it is necessary and sufficient that the set N_I' should contain the set N_I. In other words: the partly ordered set of ideals of the ring $C(S)$ with the relationship of inclusion as the ordering relationship is isomorphic to the partly ordered set of closed subsets of the space S ordered in the opposite manner.

Hence we come easily to the following

Theorem III. *In order that two metrisable compact spaces S and S' should be homeomorphic, it is necessary and sufficient that the rings $C(S)$ and $C(S')$ should be continuously isomorphic.*

Proof. The necessity of the condition is clear: the homeomorphism between the spaces S and S' automatically generates a continuous isomorphism between the rings $C(S)$ and $C(S')$.

Let us now prove that the condition is also sufficient. If the rings $C(S)$ and $C(S')$ are continuously isomorphic, the partly ordered sets

of their ideals are also isomorphic, and consequently, so are the partly ordered sets of closed subsets of the spaces S and S'. This last isomorphism, however, implies the homeomorphism of the spaces S and S'.

In considering the subrings of the ring $C(S)$, we shall confine ourselves to subrings C' satisfying the following conditions:

1) C' is topologically closed;

2) if C' contains a function $\alpha(x)$, it also contains all functions of the type $k\alpha(x)$, where k is an arbitrary complex number;

3) if C' contains a function $\alpha(x)$, it also contains the conjugated function $\overline{\alpha(x)}$.

In what follows under subrings of the ring $C(S)$ we shall understand only subrings satisfying the conditions 1)—3).

Two points x and y of the space S shall be called conjugated with respect to the subring C', if for every function α from C' we have

$$\alpha(x) = \alpha(y).$$

It is easily seen that the whole space S falls into classes of elements conjugated with respect to C' in such a way, that the elements of one and the same class are conjugated and the elements belonging to different classes are not conjugated. Further, every class X of conjugated elements is a closed set and the decomposition

$$S = \sum X$$

of the space S is a continuous decomposition in the sense of P. Alexandroff [1].

Theorem IV. *Every subring C' of the ring $C(S)$ containing the unity coincides with the set of all functions from $C(S)$, constant on each set X of the continuous decomposition*

$$S = \sum X$$

of the space S into classes of elements conjugated with respect to the subring C'.

Before proving the general Theorem IV, it is convenient to prove the following proposition which is a special case of Theorem IV:

Theorem IV_{bis}. *If the subring C° of the ring $C(S)$ contains the unity and if no two different points x and y of the space S are conjugated with respect to C°, then C° coincides with the whole ring $C(S)$.*

The proof of Theorem IV_{bis} will be given elsewhere. If we take Theorem IV_{bis} for granted, Theorem IV may be proved as follows.

Consider the subring C' satisfying the conditions of the theorem and the continuous decomposition

$$S = \sum X$$

generated by it. As is known [1], the continuous decomposition $S = \sum X$ generates a continuous mapping of the space S on a quite definite space S^*, the elements of which are the sets X. Moreover, if the space S is metrisable and compact, the space S^* will be also metrisable and compact, and every element x of the space S is mapped onto that element

$$X = \varphi(x)$$

of the space S^*, in which it is contained.

It is easily seen that every function $\alpha(x)$ from $C(S)$, constant on each of the sets X, may be represented in the form

$$\alpha(x) = \alpha^*[\varphi(x)],$$

where α^* is a certain quite definite function from $C(S^*)$. The correspondence between α^* and α establishes a continuous isomorphism between the ring $C(S^*)$ and the ring C_1 of all functions from $C(S)$, constant on each of the sets X. The ring C', with which we are concerned, is the subring C_1. In virtue of the established isomorphism between C_1 and $C(S^*)$, to the subring C' corresponds the subring C° of the ring $C(S^*)$, satisfying all conditions of Theorem IV_{bis}. Consequently, C° coincides with $C(S^*)$ and C' coincides with C_1, q. e. d.

By Theorem IV each subring C' of the ring $C(S)$ containing the unity is uniquely determined by the continuous decomposition $S = \sum X$ of the space S generated by it. Since, conversely, to every continuous decomposition of the space S into closed sets corresponds its own (and generating it) subring C' of all functions constant on each of the sets of the decomposition, there arises a one-to-one correspondence between the subrings of the ring $C(S)$ containing the unity and the continuous decompositions of the space S. Moreover, the subring C', corresponding to the decomposition $S = \sum X$, is continuously isomorphic to the ring $C(S^*)$ of the induced space S^* of the decomposition $S = \sum X$.

In conclusion, I should point out that Theorems I and II were obtained in the case when the space S is the segment $[0; 1]$ independently by V. Ditkin. The problems solved in this note were proposed to me by I. Gelfand. For this and for the constant interest which he took in my investigations I express him my sincere gratitude.

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MATHEMATICS

ON RINGS OF CONTINUOUS FUNCTIONS ON TOPOLOGICAL SPACES

By I. GELFAND and A. KOLMOGOROFF

(Communicated by I. M. Vinogradow, Member of the Academy, 17. XI. 1938)

The present note deals with the same subject as the investigations by M. Stone (2) and the note of G. Šilov published above. In difference from this last note, we consider a ring of continuous functions defined on a certain topological space as a purely algebraical formation, without introducing in it any topological relations. It turns out that in the case of bicompact spaces considered by M. Stone, as well as in considerably more general cases, the algebraical structure of the ring of continuous functions already defines the topological space up to a homeomorphism.

For any topological space S we shall consider two rings: the ring $C(S)$ of all real continuous functions defined on S and the ring $C'(S)$ of all bounded functions from $C(S)$.

It is natural that in studying these rings we should confine ourselves to the case of totally regular spaces S. This may be justified by the fact that the rings $C(S)$ and $C'(S)$ of an arbitrary topological space S are respectively isomorphic to the rings $C(\rho S)$ and $C'(\rho S)$ of a definite totally regular space ρS introduced by E. Čech (1), which is connected with S in a unique manner. Together with the space ρS E. Čech introduces a certain continuous mapping $y = \rho(x)$ of the space S on the space ρS, which possesses the following property: the real continuous functions on S coincide with functions of the type $f(x) = \varphi[\rho(x)]$, where $\varphi(y)$ is a continuous real function on the space ρS. Hence it follows that the ring $C(S)$ is isomorphic to the ring $C(\rho S)$ and the ring $C'(S)$— to the ring $C'(\rho S)$.

Thus, we may suppose that in what follows the original space S is totally regular (in this case ρS coincides with S).

We shall say that an ideal of the ring C is maximum, if it does not coincide with the whole ring and is not contained in any greater ideal except the ring C itself. Let us form from the set γ of maximum ideals of the ring C a topological space by means of the following definition: the maximum ideal α is called a point of contact of the set $\mathfrak{M}$ of maximum ideals, if it contains the intersection of all maximum ideals contained in $\mathfrak{M}$. It is easily verified that for any ring C this definition of points of contact makes the set γ to a space of the type T_1 (according to the terminology of Alexandroff-Hopf, Topologie, 1). Observe

that this method of introducing topology in the set of maximum ideals was already used by M. H. Stone (²).

We shall denote the space γ corresponding to the ring $C(S)$ by $\gamma(S)$ and the space γ corresponding to the ring $C'(S)$ by $\gamma'(S)$.

1. The Case of a Bicompact Space S

If the space S is bicompact, the rings $C(S)$ and $C'(S)$ coincide (all continuous functions on S are bounded) and it becomes unnecessary to consider them separately.

Theorem I. *If the space S is bicompact, it is homeomorphic to the space $\gamma(S)$.*

Before proving this theorem, let us establish the following

Lemma. For any ideal A of the ring $C(S)$ not coinciding with the whole ring there exists a point a of the space S, at which all functions belonging to A vanish.

Proof of the Lemma. Assume that the assertion is false. Then for every point ξ of the space S we can find a function $f_\xi(x)$ from A, which does not vanish at the point ξ. The function $f_\xi(x)$ is different from zero in a certain neighbourhood $U(\xi)$ of the point ξ. By Borel-Lebesgue's theorem we can choose a finite set of points,

$$\xi_1, \xi_2, \ldots, \xi_n,$$

so that the neighbourhoods

$$U(\xi_1), U(\xi_2), \ldots, U(\xi_n)$$

cover the whole space S. The function

$$\varphi(x) = f_{\xi_1}^2(x) + f_{\xi_2}^2(x) + \ldots + f_{\xi_n}^2(x)$$

belongs to the ideal A and is everywhere different from zero. Hence every function $f(x)$ of the ring $C(S)$ may be represented in the form $f(x) = \psi(x)\varphi(x)$, and, consequently, also belongs to A, contrary to assumption. The obtained contradiction proves the lemma.

Proof of Theorem I. From the lemma just proved it follows that any ideal A not coinciding with the whole ring $C(S)$ is contained in a certain ideal $I(a)$, consisting of all fnnctions vanishing at the point a. Consequently, only ideals of this last type can be maximal. It is also plain that any ideal of the type $I(a)$ is in fact maximal. In virtue of the total regularity of the space S the ideals $I(a)$ and $I(a')$, corresponding to two different points a and a', are different: there exists a function from $C(S)$ vanishing at the point a and different from zero at the point a'. Thus, correlating to every point a of the space S the maximum ideal $I(a)$, we obtain a one-to-one correspondence between the space S and the set $\gamma(S)$ of maximum ideals of the ring $C(S)$.

We have now to prove that the correspondence obtained between S and $\gamma(S)$ is a homeomorphism. Suppose that to a set M of points of the space S corresponds the set $\mathfrak{M}$ of maximum ideals. If a point a is a point of contact of the set M, all functions contained in the intersection of all ideals from $\mathfrak{M}$, i. e. vanishing at all points of the set M, vanish also at the point a, i. e. belong to the ideal $I(a)$. According to definition, this means, however, that $I(a)$ is a point of contact of the set $\mathfrak{M}$. Conversely, if a point a is not a point of contact for M, then, in virtue of the total regularity of S, we may construct a function $f(x)$ from $C(S)$, vanishing on M and different from zero at the point a; this function

belongs to the intersection of all ideals from $\mathfrak{M}$, but does not belong to $I(a)$; consequently, in this case $I(a)$ will not be a point of contact for $\mathfrak{M}$. The homeomorphism between the spaces S and $\gamma(S)$ is thus proved.

Theorem II. *In order that two bicompact spaces S and S_1 should be homeomorphic, it is necessary and sufficient that the rings $C(S)$ and $C(S_1)$ should be algebraically isomorphic.*

Proof. The necessity of the condition is obvious: any homeomorphism between S and S_1 automatically generates an isomorphic mapping of the ring $C(S)$ on the ring $C(S_1)$. That the condition is also sufficient, follows from Theorem 1: from the isomorphism of $C(S)$ and $C(S_1)$ plainly follows the homeomorphism of the spaces $\gamma(S)$ and $\gamma(S_1)$; but these last are, by Theorem I, homeomorphic to the spaces S and S_1.

2. The Ring $C'(S)$ in the General Case

The question of the structure of the ring $C'(S)$ of an arbitrary totally regular space S may be reduced to the case of a bicompact space S. To do this we use the space βS introduced by E. Čech. According to E. Čech, to every totally regular space S corresponds a space βS uniquely determined up to a homeomorphism and characterized by the following conditions: 1) βS is bicompact; 2) βS contains S; 3) S is dense on βS; 4) every continuous bounded real function defined on S may be continued into βS, the continuity being preserved.

Since S is dense on βS, the continuation of continuous functions from S into βS is defined uniquely. Thus, there arises a one-to-one correspondence between bounded continued functions defined on S and on βS. It is easily seen that in virtue of this correspondence the ring $C'(S)$ is isomorphically mapped on the ring $C'(\beta S)$. We have thus proved the following

Theorem III′. *The ring $C'(S)$ is isomorphic to the ring $C'(\beta S)$* [or, which is the same, to the ring $C(\beta S)$].

From Theorems I and III′ follows

Theorem IV′. *The space $\gamma'(S)$ is homeomorphic to the space βS.*

In virtue of Theorem III′ it is clear that two non-homeomorphic spaces S and S_1 may have isomorphic rings $C'(S)$ and $C'(S_1)$. To see this it is sufficient to take for S any non-bicompact space and put $S_1 = \beta S$; then, by Theorem III′, the ring $C'(S)$ will be isomorphic to the ring $C'(S_1)$. We have, however, the following

Theorem V′. *In order that the spaces S and S_1 satisfying the first axiom of enumerability should be homeomorphic, it is necessary and sufficient that the rings $C'(S)$ and $C'(S_1)$ should be algebraically isomorphic.*

Proof. That the condition is sufficient, may be proved in the same way, as in the case of Theorem I. The necessity follows from the fact, established by E. Čech, that for two spaces S and S_1 satisfying the first axiom of enumerability the homeomorphism of βS and βS_1 implies the homeomorphism of the original spaces S and S_1. Indeed, from the isomorphism of $C'(S)$ and $C'(S_1)$ follows the homeomorphism of $\gamma'(S)$ and $\gamma'(S_1)$, i. e., by Theorem IV′, the homeomorphism of βS and βS_1 and, consequently, in virtue of the just mentioned result of E. Čech, also the homeomorphism between S and S_1.

3. The Ring $C(S)$ in the General Case

For the ring $C(S)$ the theorem analogous to Theorem III′ concerning the ring $C'(S)$ is not true. For instance, if S is the space of integers

(i. e. enumerable space of isolated points) and $S_1 = \beta S$, then $C(S)$ and $C(S_1)$ are not isomorphic (we omit the proof of this fact). Theorems analogous to Theorems IV′ and V′ remain, however, valid.

Theorem IV. *The space $\gamma(S)$ is homeomorphic to the space βS.*

Theorem V. *In order that two spaces S and S_1 satisfying the first axiom of enumerability should be homeomorphic, it is necessary and sufficient that the rings $C(S)$ and $C(S_1)$ should be algebraically isomorphic.*

Theorem V may be deduced from Theorem IV in the same way as Theorem V′ was deduced from Theorem IV′. Hence we have only to prove Theorem IV.

Proof of Theorem IV. To every function $f(x)$ from $C(S)$ we let correspond the function $f_1(x) = \frac{2}{\pi} \operatorname{arctg} [f(x)]$. The function $f_1(x)$ is continuous and bounded on S. Consequently, it may be continued with preservation of continuity into the space βS. This enables us to expand, in a certain sense, the definition of the function $f(x)$ to the whole space βS. Namely, for any point of the set $\beta S - S$ we put

$$f(x) = \operatorname{tg}\left[\frac{\pi}{2} f_1(x)\right], \qquad \text{if} \quad |f_1(x)| \neq 1;$$

$$f(x) = +\infty, \qquad \text{if} \quad f_1(x) = +1;$$

$$f(x) = -\infty, \qquad \text{if} \quad f_1(x) = -1.$$

When the functions $f(x)$ are thus extended to the whole space βS, the proof of Theorem V proceeds, with certain complications, on the lines of the proof of Theorem I. We state here only the fundamental points of this proof. In the first place we establish the following

Lemma 1. For any ideal A of the ring $C(S)$, different from the whole ring, there exists a point a of the space βS, at which all functions belonging to A vanish.

Any point a of the space βS defines the ideal $I(a)$ of the ring $C(S)$ consisting of all functions $f(x)$ of this ring, for which all products

$$\varphi(x) = \psi(x) f(x)$$

with factors $\psi(x)$ from $C(S)$ vanish at the point a (if the point a belongs to $\beta S - S$, under $\varphi(a)$ we understand the value in the sense defined above).

On ground of Lemma 1 we next prove that all ideals of the type $I(a)$ are maximal and that there are no other maximum ideals in the ring $C(S)$.

Lemma 2. If the point a belongs to S, the ideal $I(a)$ consists of all functions $f(x)$ of the ring $C(S)$, vanishing at the point a. If the point a belongs to $\beta S - S$, the ideal $I(a)$ consists of all functions $f(x)$ of the ring $C(S)$, vanishing on the set N_f of points of the original space S having a for its point of contact.

We shall omit the proof of Lemma 2. From this lemma it follows that the ideals $I(a)$ and $I(a')$, corresponding to two different points a and a', are different. Thus, we can establish a one-to-one correspondence between the points a of the space βS and the maximum ideals $I(a)$. Similarly as in the case of Theorem I we prove that this correspondence between βS and $\gamma(S)$ is a homeomorphism.

Remark. From the isomorphism of the rings $C(S)$ and $C(S_1)$ follows (by Theorem IV) the homeomorphism of the spaces βS and βS_1.

and consequently (by Theorem III'), also the isomorphism of the rings $C'(S)$ and $C'(S_1)$. But, conversely, from the isomorphism of the rings $C'(S)$ and $C'(S_1)$ the isomorphism of the rings $C(S)$ and $C(S_1)$, generally speaking, does not follow. For instance, if S is the space of integers and $S_1 = \beta S$, the rings $C'(S)$ and $C'(S_1)$ are isomorphic, but the rings $C(S)$ and $C(S_1)$ are not. Thus, the ring $C(S)$ provides us with a more elastic means of investigation of the topological properties of the space S than the ring $C'(S)$. However, there still exist spaces S and S_1 with isomorphic rings $C(S)$ and $C(S_1)$, which are not homeomorphic to each other. Such are, for instance, the space S of transfinite numbers $< \Omega$ and the space S_1 of transfinite numbers $\leqslant \Omega$, taken with limiting relations defined in them in the usual manner.

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MATHEMATISCHE PHYSIK

ZUR THEORIE DER GITTER. II

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Bevor wir zu dem Beispiel $b = \frac{a}{2}$ unserer Arbeit (1) übergehen (es sei bemerkt, dass es uns gelungen ist dasselbe vollständig zu lösen), wollen wir den allgemeinen Fall betrachten, erstens, weil es möglich ist die Gleichungen für die Koeffizienten aufzustellen und, zweitens, weil der Übergang zu $\lambda \to 0$ resp. zu $\lambda \to \infty$ hier sehr leicht, aber allgemeiner auszuführen ist.

Zu diesem Zweck setzen wir in (14) und (15) unserer Arbeit (1)

$$B_n \beta_n = D_n; \quad n \geq 0, \tag{1}$$

und erhalten dann, statt derselben,

$$H_0(y) = \sum_{n=0}^{\infty} \frac{D_n}{\beta_n} \cos \frac{\pi n y}{a} \tag{2}$$

und

$$N_0(y) = \sum_{n=0}^{\infty} D_n \cos \frac{\pi n y}{a}. \tag{3}$$

Im folgenden unterscheiden wir zwei Fälle:
Fall I — die elektrische Kraft ist parallel der Spalte und
Fall II — die elektrische Kraft ist senkrecht zu denselben.

Fall I. Denselben haben wir in unserer Arbeit (1) berücksichtigt und erhalten aus (3)

$$D_0 = \frac{a}{a-b} \cdot \frac{1}{\pi} \sum_{n=1}^{\infty} \frac{D_n}{n} \sin \frac{\pi n b}{a} \tag{4}$$

und, wegen (4),

$$D_m = \frac{2a}{a-b} \frac{\sin \frac{\pi m b}{a}}{\pi^2 m} \sum_{n=1}^{\infty} \frac{D_n}{n} \sin \frac{\pi n b}{a} + \frac{1}{a} \sum_{n=1}^{\infty} D_n a_{n,m}; \quad m \geq 1 \tag{5}$$

mit derselben Bedeutung der $a_{n,m}$, wie früher.

Es sind (4) und (5) für eine jede Funktion gültig, die sich in eine Fourierreihe zerlegen lässt, welche auf dem Streifen Null sein wird und in der Öffnung unbekannt [gemischtes Problem, Arbeit (1), siehe dort auch (18) und (19)].

Laut unserem Satz erhalten wir für $H_0(y)$ aus (2)

$$\frac{D_0}{\beta_0} = \frac{1}{\beta_0} - \frac{a}{b\pi} \sum_{n=1}^{\infty} \frac{D_n}{n\beta_n} \sin \frac{\pi n b}{a} \tag{6}$$

unter Beachtung, dass $\beta_0 = \frac{i}{k} = -\frac{1}{ik}$ ist. Da nun aber

$$c_{n,m} = \int_b^a \cos \frac{\pi n y}{a} \cdot \cos \frac{\pi m y}{a} \, dy \tag{7}$$

bedeutet und

$$\left.\begin{aligned} a_{n,m} &= -2c_{n,m}; \quad n \neq m \\ a_{n,n} &= -2c_{n,n} + a; \quad n = m \end{aligned}\right\} \tag{8}$$

sein wird, so ergibt sich für die D_m von $H_0(y)$

$$0 = -\frac{2a}{bm\pi^2} \sin \frac{\pi m b}{a} \sum_{n=1}^{\infty} \frac{D_n}{n\beta_n} \sin \frac{\pi n b}{a} + \frac{1}{a} \sum_{n=1}^{\infty} \frac{D_n}{\beta_n} a_{n,m}; \quad m \geqslant 1. \tag{9}$$

Zu (6) und (9) lässt sich analoges sagen wie zu (4) und (5), weshalb wir auch in unserer Arbeit (1) den Ausdruck (9) dort berücksichtigt haben.

Wir ersehen nun, dass damit den Maxwellschen Gleichungen genügt wird, es ein und dasselbe D_n, zu gleicher Zeit, den beiden Systemen (4), (5) und (6), (9) genügen muss. Deshalb ergibt uns ein Vergleich von (4) und (6), falls wir statt $a_{n,m}$ seinen Wert einsetzen:

$$1 = \frac{1}{\pi} \sum_{n=1}^{\infty} \frac{D_n}{n} \sin \frac{\pi n b}{a} \left\{ \frac{a\beta_0}{b\beta_n} + \frac{a}{a-b} \right\}. \tag{10}$$

Weiter erhalten wir aus dem Vergleich von (5) und (9), indem wir hinterher beiderseits mit

$$\left\{ \frac{a-b}{a} - \frac{\sin \frac{2\pi m b}{a}}{2\pi m} \right\} \left\{ \frac{b}{a} + \frac{\sin \frac{2\pi m b}{a}}{2\pi m} \right\}$$

multiplizieren:

$$\frac{2}{\pi^2} \frac{\sin \frac{\pi m b}{a}}{m} \sum_{n=1}^{\infty} \frac{D_n}{n} \sin \frac{\pi n b}{a} \left[\frac{b}{a-b} - \frac{a-b}{b} \frac{\beta_m}{\beta_n} + \frac{\sin \frac{2\pi m b}{a}}{2\pi m} \left(\frac{a}{a-b} + \frac{a}{b} \frac{\beta_m}{\beta_n} \right) \right] +$$

$$+ \frac{2 \cos \frac{\pi m b}{a}}{\pi} \sum_{n=1}^{\infty}{}' \frac{n D_n}{n^2 - m^2} \sin \frac{\pi n b}{a} \left[\frac{b}{a} + \frac{a-b}{a} \frac{\beta_m}{\beta_n} + \frac{\sin \frac{2\pi m b}{a}}{2\pi m} \left(1 - \frac{\beta_m}{\beta_n} \right) \right] -$$

$$- \frac{2m \sin \frac{\pi m b}{a}}{\pi} \sum_{n=1}^{\infty}{}' \frac{D_n \cos \frac{\pi n b}{a}}{n^2 - m^2} \left[\frac{b}{a} + \frac{a-b}{a} \frac{\beta_m}{\beta_n} + \right.$$

$$\left. + \frac{\sin \frac{2\pi m b}{a}}{2\pi m} \left(1 - \frac{\beta_m}{\beta_n} \right) \right] = 0; \quad m \geqslant 1. \tag{11}$$

Aus (10) und (11) bestimmen sich dann die gemeinsamen D_n, und D_0 aus (4). Hier bedeutet der Accent nach dem Summenzeichen, dass das Glied mit $n=m$ auszulassen ist.

Am Rande des Streifens muss die elektrische Kraft gleich Null sein, d. h. es wird laut (3) sein

$$0=\sum_{n=0}^{\infty} D_n \cos\frac{\pi n b}{a}, \tag{12}$$

was sich nun auch direkt beweisen lässt.

Fall II. Es bleiben auch jetzt die Gleichungen (2) und (3) bestehen, nur dass die Bedeutung der linken Seiten eine andere sein wird, da wir die gesamte magnetische Kraft, die jetzt stetig ist, gleich $u(x, y)$ gesetzt haben. Es wird deshalb jetzt $N_0(y)$ die magnetische Kraft für $x=0$ bedeuten, welche nach unserem Satze, innerhalb des Spaltes gleich $\frac{1}{c}$ sein wird, aber auf dem Streifen unbekannt. $H_0(y)$ ist proportional der elektrischen Kraft, welche auf dem Streifen gleich Null ist. Deshalb erhalten wir, analog (4) und (5) des ersten Falles, für $N_0(y)$

$$D_0-\frac{1}{c}=-\frac{a}{\pi b}\sum_{n=1}^{\infty}\frac{D_n}{n}\sin\frac{\pi n b}{a} \tag{13}$$

und

$$D_m\left\{\frac{b}{a}+\frac{\sin\frac{2\pi m b}{a}}{2\pi m}\right\}=\frac{2a\sin\frac{\pi m b}{a}}{\pi^2 m b}\sum_{n=1}^{\infty}\frac{D_n}{n}\sin\frac{\pi n b}{a}-$$

$$-\frac{2}{\pi}\cos\frac{\pi m b}{a}\sum_{n=1}^{\infty}{}'\frac{n D_n\sin\frac{\pi n b}{a}}{n^2-m^2}+\frac{2m}{\pi}\sin\frac{\pi m b}{a}\sum_{n=1}^{\infty}{}'\frac{D_n\cos\frac{\pi n b}{a}}{n^2-m^2};\quad m\geqslant 1. \tag{14}$$

Ähnliche Ausdrücke, wie (6) und (9), erhalten wir für $H_0(y)$. Ein Vergleich derselben mit (13) und (14) ergibt uns, analog (10) und (11), die Bestimmungsgleichungen der gemeinsamen D_m für den Fall II. D. h.

$$\frac{1}{c}=\frac{1}{\pi}\sum_{n=1}^{\infty}\frac{D_n}{n}\sin\frac{\pi n b}{a}\left\{\frac{\beta_0}{\beta_n}\frac{a}{a-b}+\frac{a}{b}\right\} \tag{15}$$

und

$$\frac{2}{\pi^2}\frac{\sin\frac{\pi m b}{a}}{m}\sum_{n=1}^{\infty}\frac{D_n}{n}\sin\frac{\pi n b}{a}\left[\frac{a-b}{a}-\frac{b}{a-b}\frac{\beta_m}{\beta_n}-\frac{\sin\frac{2\pi m b}{a}}{2\pi m}\left(\frac{a}{b}+\frac{a}{a-b}\frac{\beta_m}{\beta_n}\right)\right]-$$

$$-\frac{2}{\pi}\cos\frac{\pi m b}{a}\sum_{n=1}^{\infty}{}'\frac{n D_n}{n^2-m^2}\sin\frac{\pi n b}{a}\left[\frac{a-b}{a}+\frac{b}{a}\frac{\beta_m}{\beta_n}-\frac{\sin\frac{2\pi m b}{a}}{2\pi m}\left(1-\frac{\beta_m}{\beta_n}\right)\right]+$$

$$+\frac{2m}{\pi}\sin\frac{\pi m b}{a}\sum_{n=1}^{\infty}{}'\frac{D_n}{n^2-m^2}\cos\frac{\pi n b}{a}\left[\frac{a-b}{a}+\frac{b}{a}\frac{\beta_m}{\beta_n}-\right.$$

$$\left.-\frac{\sin\frac{2\pi m b}{a}}{2\pi m}\left(1-\frac{\beta_m}{\beta_n}\right)\right]=0;\quad m\geqslant 1. \tag{16}$$

Nun gehen wir zu $\lambda\to 0$ resp. zu $\lambda\to\infty$ über.

Zu diesem Zweck setzen wir in (10) und (11) (Fall I)

$$D_n = \frac{2a^2}{\pi n b\,(a-b)} \cdot \frac{\sin\frac{\pi n b}{a}}{\frac{a\beta_0}{b\beta_n} + \frac{a}{a-b}} + P_n \tag{17}$$

und in (15) und (16) den Ausdruck (Fall II)

$$D_n = \frac{2a^2}{c\,n b\,(a-b)} \frac{\sin\frac{\pi n b}{a}}{\frac{a}{a-b}\frac{\beta_0}{\beta_n} + \frac{a}{b}} + \frac{K_n}{c}. \tag{18}$$

Wir erhalten dann aus (10) und (17)

$$\sum_{n=1}^{\infty} \frac{P_n \sin\frac{\pi n b}{a}}{n} \left\{ \frac{a\beta_0}{b\beta_n} + \frac{a}{a-b} \right\} = 0 \tag{19}$$

und aus (15) und (18)

$$\sum_{n=1}^{\infty} \frac{K_n \sin\frac{\pi n b}{a}}{n} \left\{ \frac{a}{a-b}\frac{\beta_0}{\beta_n} + \frac{a}{b} \right\} = 0. \tag{20}$$

Nun lässt es sich nachweisen, dass es sein wird:

$$0 = \frac{1}{\pi m} \sin\frac{\pi m b}{a} \sum_{n=1}^{\infty}{}' \frac{\sin^2\frac{\pi n b}{a}}{n^2} \left[\frac{a}{a-b} - \frac{a}{b} + \frac{a^2}{b\,(a-b)} - \frac{\sin\frac{2\pi m b}{a}}{2\pi m} \right] +$$

$$+ \cos\frac{\pi m b}{a} \sum_{n=1}^{\infty}{}' \frac{\sin^2\frac{\pi n b}{a}}{n^2 - m^2} - m \sin\frac{\pi m b}{a} \sum_{n=1}^{\infty}{}' \frac{\sin\frac{\pi n b}{a} \cdot \cos\frac{\pi n b}{a}}{n\,(n^2 - m^2)}; \quad m \geqslant 1. \tag{21}$$

Nachdem wir in (11) den Ausdruck (17) eingesetzt haben, übertragen wir diejenigen Glieder, welche nicht von P_n abhängen, nach rechts. Diese rechte Seite lässt sich nun wegen (21) umformen. Analoges gilt für (16), nach Einsetzen des Ausdruckes (18).

Nun ist bei $\lambda \to 0$

$$\frac{\beta_m}{\beta_n} = 1 \tag{22}$$

bei beliebigem m und n. Deshalb verschwinden die rechten Seiten der oben erwähnten Ausdrücke, was wegen (19) und (20) zu $P_n = K_n = 0$ führt.

Die Ausdrücke (17) und (18) werden, abgesehen vom Faktor $\frac{1}{c}$, einander gleich und zwar gleich sein

$$D_n = \frac{2}{\pi n} \sin\frac{\pi n b}{a}. \tag{23}$$

Aus (4) und (13) erhalten wir für D_0 den gleichen Wert (wieder abgesehen vom Faktor $\frac{1}{c}$) und zwar:

$$D_0 = \frac{b}{a}. \tag{24}$$

Ohne jede Rechnerei kommen wir zu demselben Resultat (23) und (24) direkt aus (2) und (3) nach Multiplikation des ersten Ausdruckes

mit β_0. Wegen (22) werden die rechten Seiten dieser Ausdrücke, also auch die linken Seiten, einander gleich sein. Nun wissen wir aber, dass im Falle I die elektrische Kraft auf dem Streifen gleich Null sein wird, was also auch für die rechte Seite gilt. Innerhalb der Öffnung ist die magnetische Kraft gleich eins (wegen der Multiplikation mit β_0). Dies letztere gilt ebenfalls für die rechte Seite. Deshalb ist uns die Funktion, die die rechte Seite von (2) resp. (3) darstellt, längs der ganzen Periode $2a$ gegeben und wir kommen so zu dem bekannten Ausdruck

$$F(y) = \frac{b}{a} + \frac{2}{\pi} \sum_{n=1}^{\infty} \frac{\sin \frac{\pi n b}{a}}{n} \cos \frac{\pi n y}{a}, \tag{25}$$

wobei

$$F(y) = \begin{cases} 1; & |y| < b \\ 0; & |y| > b \end{cases} \tag{26}$$

sein wird.

Es wird endlich die Intensität I_n, wobei wir diejenige des direkt durchgegangenen Lichtes gleich eins setzen, sein

$$I_n = \left(\frac{a}{\pi b}\right)^2 \frac{\sin^2 \frac{\pi n b}{a}}{n^2}. \tag{27}$$

Zu demselben Resultat (27) kommt auch Rayleigh [2] bei seiner angenäherten Theorie des Lichtes.

Für $\lambda \to \infty$ haben wir

$$\frac{1}{\beta_n} = -\frac{\pi n}{a}, \quad n \geqslant 1; \quad \beta_0 = \frac{i\lambda}{2\pi}. \tag{28}$$

Deshalb verschwinden, nach Einsetzen von (17) resp. (18), die rechten Seiten von (13) resp. von (16) und wir kommen ebenfalls zu dem Resultat, dass $P_n = K_n = 0$ ist. Demnach ergibt sich, wegen (28), dass in (17) und (18) die D_n verschwinden, wie $\frac{1}{\lambda}$ bei $\lambda \to \infty$.

Also haben wir im Falle I laut (4) (bei $\lambda \to \infty$)

$$D_0 = 0 \tag{29}$$

und im Falle II laut (13)

$$D_0 = \frac{1}{c}. \tag{30}$$

Wir ersehen also aus dem vorhergehenden, dass bei $\lambda \to 0$ kein Unterschied zwischen den Fällen I und II zu beobachten sein wird.

Bei $\lambda \to \infty$ wird im Falle I, laut (29), nichts durchgelassen, während im Falle II, laut (30), die Welle ungehindert durch das Gitter hindurchgeht. Es ist selbstverständlich, dass hierbei $b > 0$ sein muss.

Wir müssen noch bemerken, dass wir die Fälle $\lambda \to 0$ und $\lambda \to \infty$ nur als Grenzfälle betrachten können, denn, laut (26), macht $F(y)$ bei $|y| = b$ einen Sprung, was der Beziehung (12) widerspricht. Im anderen Fall kommen wir zu einem statischen Zustand, was sich durch die Annahme einer unendlich langen Welle erklärt.

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MATHEMATISCHE PHYSIK

ZUR THEORIE DER GITTER. III

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Wir gehen nun zu

$$b = \frac{a}{2} \tag{1}$$

über. Zuerst untersuchen wir den Fall I.

Aus (4) und (5) unserer vorhergehenden Arbeit ergibt sich

$$D_0 = \frac{2}{\pi} \sum_{n=0}^{\infty} \frac{(-1)^n D_{2n+1}}{2n+1}, \tag{2}$$

$$D_{2m} = \frac{4}{\pi} (-1)^m \sum_{n=0}^{\infty} \frac{(-1)^n (2n+1) D_{2n+1}}{(2n+1)^2 - (2m)^2}; \quad m \geqslant 1 \tag{3}$$

und

$$D_{2m+1} = -\frac{4}{\pi} (2m+1) (-1)^m \sum_{n=0}^{\infty} \frac{(-1)^n D_{2n}}{(2n)^2 - (2m+1)^2}; \quad m \geqslant 0. \tag{4}$$

Analog erhalten wir aus (6) und (9) unser Arbeit:

$$D_0 - 1 = -\frac{2}{\pi} \sum_{n=0}^{\infty} \frac{(-1)^n D_{2n+1}}{2n+1} \frac{\beta_0}{\beta_{2n+1}}, \tag{5}$$

$$D_{2m} = -\frac{4}{\pi} (-1)^m \sum_{n=0}^{\infty} \frac{(-1)^n (2n+1) D_{2n+1}}{(2n+1)^2 - (2m)^2} \frac{\beta_{2m}}{\beta_{2n+1}}; \quad m \geqslant 1 \tag{6}$$

und

$$\left.\begin{aligned} D_{2m+1} &= \frac{4}{\pi} \frac{(-1)^m}{(2m+1)} \frac{\beta_{2m+1}}{\beta_0} + \\ &+ \frac{4}{\pi} (2m+1) (-1)^m \sum_{n=0}^{\infty} \frac{(-1)^n D_{2n}}{(2n)^2 - (2m+1)^2} \frac{\beta_{2m+1}}{\beta_{2n}}; \quad m \geqslant 0. \end{aligned}\right\} \tag{7}$$

Nun lässt es sich nachweisen, dass falls (2) und (3) richtig sind, es auch (4) existieren wird; umgekehrt, existiert (4) und (2), so ist auch (3)

richtig. Dasselbe lässt sich auch bezüglich (5) – (7) sagen. Demnach genügt es nur (2) mit (5) und (3) mit (6) zu vergleichen. Wir erhalten deshalb

$$\sum_{n=0}^{\infty} \frac{(-1)^n D_{2n+1}}{2n+1} \left\{ \frac{\beta_0}{\beta_{2n+1}} + 1 \right\} = \frac{\pi}{2} \tag{8}$$

und

$$\sum_{n=0}^{\infty} \frac{(-1)^n (2n+1) D_{2n+1}}{(2n+1)^2 - (2m)^2} \left\{ \frac{\beta_{2m}}{\beta_{2n+1}} + 1 \right\} = 0; \; m \geqslant 1. \tag{9}$$

Dies sind diejenigen Gleichungen, aus denen sich die unbekannten und gemeinsamen D_{2n+1} bestimmen lassen müssen. Wie wir sehen, wird es hierbei nur nötig sein die ungeraden D_{2n+1} zu wissen. Dann berechnen sich D_0 aus (2) und die geraden D_{2m} aus (3). Nun wird sein

$$\frac{\frac{1}{\beta_{2n+1}} + \frac{1}{\beta_{2m}}}{(2n+1)^2 - (2m)^2} = \left(\frac{\pi}{a}\right)^2 \frac{1}{\frac{1}{\beta_{2n+1}} - \frac{1}{\beta_{2m}}}. \tag{10}$$

Wir schreiben weiter

$$\frac{\beta_0}{\beta_n} = \alpha_n; \; \alpha_0 = 1 \tag{11}$$

so dass wir haben

$$\left.\begin{aligned} \alpha_n &= \sqrt{1 - \left(\frac{n\lambda}{2a}\right)^2}; \; \frac{n\lambda}{2a} < 1 \\ \alpha_n &= 0; \; \frac{n\lambda}{2a} = 1 \\ \alpha_n &= -i\sqrt{\left(\frac{n\lambda}{2a}\right)^2 - 1}; \; \frac{n\lambda}{2a} > 1 \end{aligned}\right\}. \tag{12}$$

Jetzt setzen wir

$$D_{2n+1} = \frac{(-1)^n \pi}{2} \left(\frac{2a}{\lambda}\right)^2 \frac{M_{2n+1}}{2n+1}. \tag{13}$$

Dann fliesst aus (8) und (9) das System

$$\sum_{n=0}^{\infty} \frac{M_{2n+1}}{\alpha_{2n+1} - \alpha_0} = -1 \tag{14}$$

und

$$\sum_{n=0}^{\infty} \frac{M_{2n+1}}{\alpha_{2n+1} - \alpha_{2m}} = 0; \; m \geqslant 1. \tag{15}$$

Statt (2) und (3) erhalten wir, wegen (13)

$$D_0 = \left(\frac{2a}{\lambda}\right)^2 \sum_{n=0}^{\infty} \frac{M_{2n+1}}{(2n+1)^2} \tag{16}$$

und

$$D_{2m} = (-1)^m 2 \left(\frac{2a}{\lambda}\right)^2 \sum_{n=0}^{\infty} \frac{M_{2n+1}}{(2n+1)^2 - (2m)^2}; \; m \geqslant 1. \tag{17}$$

Wir gehen jetzt zu dem Fall II über. Derselbe lässt sich nun schnell erledigen. Wir setzen hierbei

$$D_n = \frac{D'_n}{c}. \tag{18}$$

Das System (14), (15) bleibt bestehen und statt (16) und (17) haben wir

$$D'_0 = 1 - \left(\frac{2a}{\lambda}\right)^2 \sum_{n=0}^{\infty} \frac{M_{2n+1}}{(2n+1)^2} \tag{19}$$

und

$$D'_{2m} = -(-1)^m 2 \left(\frac{2a}{\lambda}\right)^2 \sum_{n=0}^{\infty} \frac{M_{2n+1}}{(2n+1)^2 - (2m)^2}; \quad m \geqslant 1. \tag{20}$$

Setzen wir noch in (13) D'_{2n+1} statt D_{2n+1}, so bestimmen sich dann hieraus die ersteren Grössen.

Jetzt wenden wir uns zu dem System (14) und (15) und suchen nun die Lösung davon. Hierzu führen wir die Funktion $M_{2n+1,s}$ ein, welche die Lösung desselben Systems darstellt, aber in dem Fall, wo nur s Gleichungen mit s Unbekannten angenommen ist. Wir betrachten also ein quadratisches Gleichungssystem, in welchem immer die erste Gleichung [also (14)] unhomogen sein wird und alle anderen homogen. Die folgenden Glieder nehmen wir hierbei gleich der Null an. Dann haben wir

$$M_{2n+1,s} = -\frac{(a_1 - a_0)(-1)^n}{\prod_{p=0}^{p=n-1} (a_{2n+1} - a_{2p+1}) \cdot \prod_{p=1+n}^{p=s-1} (a_{2p+1} - a_{2n+1})} \cdot \prod_{p=1}^{p=s-1} \frac{(a_{2p+1} - a_0)(a_{2n+1} - a_{2p})}{(a_0 - a_{2p})}. \tag{21}$$

Hierzu müssen wir bemerken, dass im ersten Produkt im Nenner eins für $n=0$ zu setzen ist. Im zweiten Produkt im Nenner ist ebenfalls eins zu setzen, solange $s-1 < 1+n$.

Bei $s=\infty$ geht $M_{2n+1,s}$ in M_{2n+1} über, d. h. in die gesuchte Lösung. Man kann bei jedem endlichen n die Konvergenz (absolute) des unendlichen Produktes, aus welchem M_{2n+1} besteht, zeigen. Der Wert von $M_{2n+1,s}$ des letzten Gliedes unseres quadratischen Schemas entspricht $s=n+1$. Gehen wir hierbei zu $n=\infty$ über, so kann man wieder zeigen, dass hierbei $M_{2n+1,n+1}$ auch beschränkt bleibt.

Aus (12) unserer Arbeit folgt, dass die elektrische Kraft im Falle I überall stetig ist ($x=0$). Es ist klar, dass hierbei die magnetische Kraft, d. h. die Gleichung (2) derselben Arbeit, am Rande des Streifens einen Sprung erleidet. Das Umgekehrte beobachten wir im Falle II. Wir bezeichnen nun

$$M_{2n+1} = s'_{2n+1} + i s''_{2n+1}, \tag{22}$$

$$\sum_{n=0}^{\infty} \frac{M_{2n+1}}{(2n+1)^2 - (2m)^2} = L'_{2m} + i L''_{2m} \tag{23}$$

und

$$\sum_{n=0}^{\infty} \frac{M_{2n+1}}{(2n+1)^2} = S' + iS''. \tag{24}$$

Dann erhalten wir für die Intensitäten I_n, falls wir diejenige des direkt durchgegangenen Lichtes gleich eins annehmen, d. h. $I_0 = 1$ setzen, die folgenden Werte. Für den Fall I

$$I_{2n+1} = \left\{\frac{\pi}{4(2n+1)}\right\}^2 \frac{s'^2_{2n+1} + s''^2_{2n+1}}{S'^2 + S''^2}; \quad I_{2n} = \frac{L'^2_{2n} + L''^2_{2n}}{S'^2 + S''^2} \tag{25}$$

und für den Fall II

$$I_{2n+1} = \left\{\frac{\pi}{4(2n+1)}\right\}^2 \frac{s'^2_{2n+1} + s''^2_{2n+1}}{\left\{\left(\frac{\lambda}{2a}\right)^2 - S'\right\}^2 + S''^2}; \quad I_{2n} = \frac{L'^2_{2n} + L''^2_{2n}}{\left\{\left(\frac{\lambda}{2a}\right)^2 - S'\right\}^2 + S''^2}; \tag{26}$$

wobei in beiden Fällen $I_0 = 1$ zu setzen ist.

Als Zahlenbeispiel nehmen wir an:

$$\lambda = 5 \cdot 10^{-5} \text{ cm}; \quad 2a = 2 \cdot 10^{-4} \text{ cm}; \quad \frac{\lambda}{2a} = 0.25 = \frac{1}{4}$$

und erhalten laut dem Vorhergehenden

I_0	I_1	I_2	I_3	
1	0.4515	0.0513	0.1214;	Fall I
1	0.4174	0.0474	0.1122;	Fall II
1	0.4053	0.0000	0.0450;	$\lambda \to 0$.

Die dritte Zeile ist aus (27) unserer Arbeit berechnet, unter Berücksichtigung der Beziehung (1).

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PHYSICS

A NEW METHOD OF HELIUM LIQUEFACTION BY MEANS OF THE JOULE-THOMSON EFFECT

By I. L. ZELMANOV

(Communicated by A. F. Joffe, Member of the Academy, 22. XI. 1938)

An analysis of the question as to the choice of the most advantageous pressure for the liquefaction of helium in the course of throttling has led the author to the conclusion that the cause of the discrepancies between the practically most profitable pressure and that which should be expected on the basis of the $T—S$ diagram of Keesom and Houthoff (1) may lie in the impossibility of a complete heat-exchange, connected with the shape of the isenthalpies in the region of $T—S$ diagram, bordering the boundary curve of the liquid (2). In fact, if the shape of the isenthalpies in the region under consideration is such as presented in Fig. 1, then for pressures, exceeding a certain value, the complete heat-exchange between the compressed helium and the vapours of the liquid boiling under normal pressure is evidently impossible. The question of the most profitable pressure is thus decided not only by the position of the inversion point of the Joule-Thomson effect of the temperature of the compressed gas entering the heat-exchanger, but also by the shape of the isenthalpies at the temperatures of the escape from the heat-exchanger, since this shape determines the possibility of heat-exchange under the conditions chosen.

In order to find out the importance of this circumstance for the liquefaction of helium, we have calculated the enthalpies of the compressed helium at the temperature of boiling helium and the relative quantities of the liquid, which are obtained by means of throttling till 1 atm. of helium compressed and cooled down to boiling temperature (3). These relative quantities of the liquid (η, see table) are at the same time the limiting liquefaction coefficients for the pressure considered, since it is evident that the vapours of boiling helium cannot cool the compressed gas below the boiling temperature.

p	i	η	Note
10	0.93	0.81	If we extrapolate the values of the table, we shall find that at pressures higher than 40 atm. the limiting liquefaction coefficient becomes zero
15	1.57	0.68	
20	2.22	0.55	
25	2.86	0.42	
30	3.50	0.29	

Practically in order to make the heat-exchange possible, it is necessary to keep on the cold end of the heat-exchanger a temperature difference of the order 0.5—1°, so that the values of the limiting liquefaction coefficients be accordingly lower.

The heat-exchange can, however, be rendered possible by inserting into the usual scheme of the liquefier an additional throttle-valve (Fig. 2, *a*), so that at a certain intermediate temperature the pressure can be changed from the initial value to some intermediate one, the course of the process corresponding to the fat line in Fig. 1. Under such conditions the difficulties connected with the impossibility of heat-exchange disappear, and the application of pressures ranging up to the pressure of the inversion at the initial temperature becomes possible. Since, however, it is difficult to place the additional throttle-valve beforehand in the most advantageous way, it is preferable to resort to a somewhat more complicated scheme (Fig. 2, *b*). By varying the quantities of the gas, passing through the valves *1* and *2*, one can secure the type of process chosen beforehand. The application of a pressure of 60 atm. permits of a fairly efficient liquefaction of helium preliminary cooled down by hydrogen, boiling under atmospheric pressure. The actually attainable value of the liquefaction coefficient is equal to 9—10%, the expenditure of hydrogen—1.2 litre per litre of liquid helium.

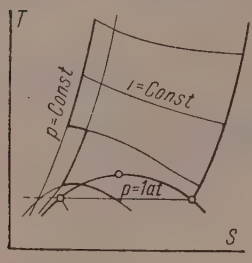

Fig. 1.

In order to check the correctness of the principles advanced above, a small liquefier was designed and built according to the scheme of Fig. 2, *b*. For sake of simplicity the liquefier was constructed without heat-exchanger for the temperatures above the boiling temperature of nitrogen, so that the experiment was started from the temperature of liquid nitrogen.

The tests of the liquefier carried out (20—27.X) at the Kriogen Laboratorium of the Ukrainian Physico-Technical Institute have completely supported the initial calculation. The initial pressure was equal to 60 atm. The intermediate one 17 atm., the intermediate temperatute$\sim$12° K.

In spite of the relatively large losses connected with the small productivity (60 cm^3/hour), a liquefaction coefficient 7.5% was obtained with an expenditure of hydrogen close to the calculated one.

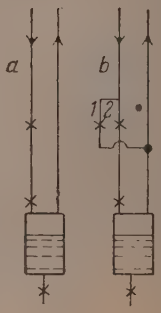

Fig. 2.

At present a liquefier is being constructed with a higher productivity (2 litre/hour). This liquefier will work with an inner hydrogen cycle. Its calculated data are as follows: the expenditure of compressed helium 8 m^3 per litre, of hydrogen compressed to 175 atm.—4.5 m^3 litre and, finally, the expenditure of liquid nitrogen—1—1.5 litre litre. It is possible that the application of pressures larger than 60 atm. will lead to a further increase of the liquefaction coefficient. This question will be investigated in the near future.

In conclusion the author wishes to express his deep gratitude to the staff of the Kriogen Laboratorium of the Ukrainian Physico-Technical Institute for their kind assistance in the tests of the model of the liquefier.

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PHYSICS

IRON CRYSTAL ORIENTATION ON MAGNETITE REDUCED BY HYDROGEN

By N. BUYNOV, M. JURAVLEVA, A. KOMAR and G. ČUFAROV

(Communicated by S. I. Wawilow, Member of the Academy, 15. XI. 1938)

It is known from many researches on oxydizing single crystals of iron, copper and barium, that the oxides obtain a crystallographic orientation with respect to the mother metal crystal ([1], [2], [3]). It has been found out, too, that when one of the components of the solid binary metallic solution is being removed ([4],[5]) and when the zeolithes are being transformed ([6]), the crystals retain their original shape.

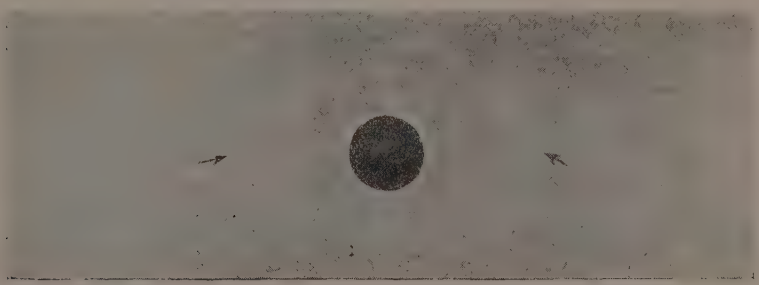

Fig. 1.

Hence it appeared to us of some interest to investigate thoroughly the orientation of iron with respect to magnetite. There are literary data available as to the possibility of the aforesaid orientation of iron ([7]).

One of the authors of this article was, besides, highly interested in the effect of orientation on the rate of reduction of the magnetite. These have been the premises of the present study.

Iron some thousandths of a mm thick has been obtained on reducing magnetite in the hydrogen at 500° C. The reduction was effected above and below the quaternary point. The methods of reduction have been described by one of the authors ([8]). The orientation of iron was investigeted by means of X-ray. We obtained both Laue and rotation photograms. The rotation photogram of a single crystal of magnetite is shown in Fig. 1. The crystal of magnetite was rotating parallel to the axis [001]. On the photogram of Fig. 1 are clearly

visible the spots due to the single crystal of iron. Those spots are denoted by arrows. The orientation of iron is of such a nature, that the axis [001] of the iron crystal is parallel to the rotation axis and, consequently, to the axis [001] of magnetite. Thus, we obtained a single iron crystal from a single magnetite crystal. When the time of reduction is prolonged, the orientation gradually disappears, which may be seen from the iron Debye-photogram combined with the magnetite rotation photogram obtained.

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PHYSICS

ON THE TIME OF THE RESTORATION OF CONTROLLABILITY IN A GAS DISCHARGE

By V. L. GRANOVSKY

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In the well-known articles by Hull and Langmuir ([1],[2]) containing an account of the basic principles of electrostatic control by gas discharge, the significance of the deionization process has been outlined along with other problems. In a stationary discharge the negatively charged grid is surrounded by an ion envelope in which the whole field of the grid is concentrated. With ordinary values for the density of the discharge current and tension in the grid, the thickness of the ion film is small, as compared with the cross-section of the meshes of the grid; the grid does not affect the discharge current. Upon discontinuation of the discharge, the density of ions falls and the thickness of the ion layer around the grid increases. Upon the expiration of a certain time, the thickness of the layer of ions, x, becomes equal to R—the radius of a mesh in the grid. Then the grid becomes again capable of locking up the discharge. This time of restoration of controllability [Freiwerdezeit, according to Hermann ([3])] depends upon the original thickness of the layer of ions, and consequently, upon the amperage of discharge current i, pressure of gas p, tensity of grid U_g with respect to plasm; further, upon the rate of deionization, and consequently, upon the general configuration of the solid surfaces of the tube and once more upon the pressure of gas; finally, upon the radius of the meshes of the grid R. For this time-interval*, without providing its deduction, Hull and Langmuir have given the following formula, which they have characterized as «a semi-empirical one»:

$$\theta = \frac{0.0012\, pi^{0.7}}{U_g^{\frac{3}{2}} \cdot d}, \qquad (1)$$

where d is the distance from the grid to the anode. This formula was subsequently reprinted in books and abstracts on ionic apparatus without any analysis whatsoever.

* Hull and Langmuir called it «the time of deionization»; yet at present we make a distinction between these two concepts. The time of deionization is such an interval of time during which the density of ions falls below a certain limit; unlike the time of the restoration of controllability, it depends but slightly upon the cross-section of the wires of the grid. Neither does it hinge upon the breadth of the meshes between these wires or upon the tension in the grid (provided the latter is not too big). Hull and Langmuir's formula (1) refers precisely to the time of restoration of controllability.

At the same time simple calculation resting upon the physical conceptions developed by Hull and Langmuir, as well as upon certain firmly established facts, leads to an altogether different formula.

Let us take our starting point in the following conditions:

1. Only ionic current runs onto the grid: the electronic component of the current is equal to zero. Thereby the assumption is made that $U_g < 0$ and is not very small in regard of its absolute magnitude.

2. The flux of ions running towards the grid is restricted by the space discharge. The thickness of the ion layer about grid x is determined by the density of the ion current j. With low pressure, when the length of the free path of an ion $\lambda_i \gg x$, resort may be had to the formula given by Langmuir and Blodgett [3] for a cylindric configuration:

$$j = \frac{5.41 \cdot 10^{-8} U_g^{\frac{3}{2}}}{\sqrt{M}\, r^2 \beta^2 \left(\frac{r+x}{r}\right)}, \qquad (2)$$

where r is the radius of the grid wires, M — molecular weight of the gas, β^2 — a definite function. If the grid is a disc with perforated apertures or if it has the shape of a funnel, it will be possible to make use of the same formula (2), only, instead of $\beta^2\left(\frac{r+x}{r}\right)$, there should stand another function $\beta_1^2\left(\frac{R-x}{R}\right)$ (the case of an internal cathode), R being the cross-section of a mesh in the grid. With higher pressures instead of (2) we should use one of the formulae deduced by McCurdy [4] or Aston which take due account of gas pressure p. We may then write in general terms:

$$j = \frac{c U_g^{\alpha}}{p^{\gamma} f(x)}. \qquad (3)$$

Here in Langmuir's case $\alpha = \frac{3}{2}$, $\gamma = 0$; in McCurdy's case $\alpha = \frac{3}{2}$, $\gamma = \frac{1}{2}$; in Aston's case $\alpha = 2$, $\gamma = 1$.

Function $f(x)$ comprises, as a parameter, the radius of the grid wires or the radius of a mesh; constant c comprises the molecular weight of the gas. In formulae (2) and (3) we reject correction $\left(1 + 0.0247\sqrt{\frac{T_i}{U_g}}\right)$, accounting for the initial velocity of ions, since we are not provided with data on the latter. This inaccuracy is, however, apt to have any effect only with small values of U_g.

3. The density of ionic current declines in the stage of deionization according to the exponential law

$$j = j_0 e^{-\frac{t}{\tau}}. \qquad (4)$$

At low pressures the rate of the disappearance of the main bulk of ions is represented with a sufficiently high degree of accuracy in the terms of this law [5—10]. Generally speaking, the fall in the ionic current proceeds according to a more complex law [10]. Yet, under the conditions existing in practice, e. g. in thyratron with mercury vapours, deviations from formula (4) become noticeable only at the end of deionization. This inexactitude — just as the preceding one — can have its effect only with small U_g.

4. The course of deionization is independent of applied grid tensions U_g. This statement has also been established on the ground of experimental evidence ([6],[7],[10],[11]). In the latest experiments of the author it was found that only tensions of the order of 1 kV and higher appreciably accelerate the course of deionization of mercury vapour pressures up to several score μ.

5. The controllability of the discharge is recovered, when $x = R$.

These assumptions are sufficient for drawing conclusion. From formulae (3) and (4) we find:

$$\frac{cU_g^2}{p^\gamma f(x)} = j_0 e^{-\frac{t}{\tau}}.$$

To the moment of time $t = \theta$, $x = R$; consequently,

$$\frac{cU_g^2}{p^\gamma f(R)} = j_0 e^{-\frac{\theta}{\tau}}.$$

Hence

$$\theta = \tau \ln \frac{j_0 \cdot p^\gamma f(R)}{cU_g^2}. \tag{5}$$

In particular for low pressure ($\alpha = \frac{3}{2}$, $\gamma = 0$) and a wire grid $\left[f(R) = r^2 \beta^2 \left(\frac{R}{r} \right) \right]$

$$\theta = \tau \left[16.75 + \ln \frac{j_0 \sqrt{M} r^2 \beta^2 \left(\frac{R}{r} \right)}{U_g^{\frac{3}{2}}} \right]. \tag{5'}$$

Thus, instead of the dependence of degree of θ upon i and U_g following from Hull and Langmuir's formula (1), we obtain a logarithmical dependence (5) *. The dependence upon the configuration of the electrodes and the walls is accounted for by the temporal constant of deionization τ. Dependence upon pressure also enters magnitude τ, moreover p has also an influence upon the magnitude of $j_0 \left(\frac{\partial j_0}{\partial p} > 0 \right)$ and, lastly, p enters in a direct way into formula (5).

For comparison of both formulae with experimental evidence we have now at our disposal a number of more complete and fresher data found in the works by Hermann ([7]) and Druskina and Krassilov ([12]). The dependence of θ upon U_g with different R, j_0 and p, as given in the work of Hermann, is represented in Figs 17 and 18. Out of all the curves plotted in these figures let us work out three of the most reliable ones: when $R = 1.35$ mm, $p = 6.5$ μ, $U_a = 240$ V, $i = 0.08$ A (let us denote it by a); when $R = 1.35$ mm, $p = 6.5$ μ, $U_a = 120$ V, $i = 0.8$ A (denoting by b); and when $R = 3$ mm, $p = 9.5$ μ, $U_a = 100$ V, $i = 0.8$ A (denoting by c); the rest of the curves contain either evident errors or an inadequate number of points. In working out this material, we shall take instead of U_g the potential of the grid relatively to the cathode; in the stage of deionization this ought not to give any grave error, as the cathode fall of potential has disappeared. According to Hull and Langmuir's formula (1) the graph of the dependence of lg θ upon lg U_g should be a straight line

* In the first approximation j_0 is proportional to i.

with an angular coefficient of $-\frac{3}{2}$. In Fig. 1 Hermann's data are represented on a logarithmical scale; in the figure, for comparison, a straight line is plotted corresponding to formula (1), i. e. with a slope of $-\frac{3}{2}$. It is obvious that all the three experimental curves fail altogether to agree with formula (1).

From the obtained by us formula (5), θ ought to be a linear function of $\lg U_g$. In Fig. 2 the same curves *a*, *b* and *c* are replotted on a semi-logarithmical scale ($\theta - \lg U_g$). We see that with the use of such a scale the points are superimposed on a straight line in a quite satisfactory manner, with the exception of the smallest tensions (when $|U_g| < 8$

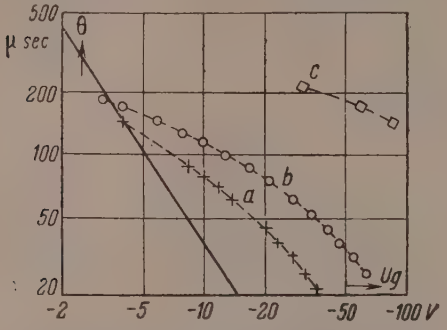

Fig. 1. Comparison of experimental data by Hermann (dotted line) with Hall and Langmuir's formula (1) (solid line).

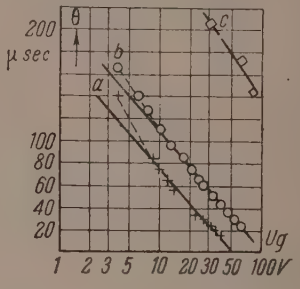

Fig. 2. Same—with formula (5).

volts), for which the premises used in the deduction of formula (5) do not hold.

Further, comparing the run of curves *a* and *b* in Fig. 2, distinguished by amperage i with equal p and R, we see that they are parallel. In his conclusion the experimenter emphasizes the fact that with change of amperage i the time of recovery changes by magnitude θ which depends but little upon U_g, while θ itself is strongly dependent upon U_g. In terms of mathematics this means that

$$\frac{\partial}{\partial U_g}\left(\frac{\partial \theta}{\partial i}\right) = \frac{\partial^2 \theta}{\partial U_g \, \partial i} \approx 0.$$

Such a result follows in fact from formula (5):

$$\frac{\partial \theta}{\partial i} = \frac{\partial \theta}{\partial j_0} \cdot \frac{\partial j_0}{\partial i} = \frac{\tau}{j_0} \cdot \frac{\partial j_0}{\partial i};$$

and in this expression τ does not at all depend upon U_g, while j_0 depends but little on this magnitude (with not very small values of U_g the grid current becomes saturation current). Per contra, this result contradicts formula (1), according to which

$$\frac{\partial^2 \theta}{\partial U_g \, \partial i} = \text{const} \, \frac{p}{i^{0.3} \, U_g^{\frac{3}{2}} \, d} \neq 0.$$

Finally, from formula (5) it follows also that, according the slope of curves $\theta - \lg U_g$, it is quite possible to calculate the temporal con-

stant of deionization, τ. Namely, we have: $-\frac{\partial\theta}{\partial \ln U_g} = \alpha\tau$. In Hermann's work are to be found direct measurements of the course of deionization under conditions corresponding to curve c (Fig. 15, curve $I-II$). From curve c we find $-\frac{\partial\theta}{\partial \ln U_g} = 64$ μsec. Assuming $\alpha = \frac{3}{2}$ (low pressure), we obtain $\tau = 43$ μsec. At the same time the direct measurements of Hermann give for the first part of curve $\tau = 49$ μsec.

In Fig. 3 are to be found data taken from the works of Druskina and Krassilov[12] which were obtained in thyratron with different values of i (150 and 225 mA), but all other things being equal. Here again we find agreement with the conclusions of formula (5) in two respects: 1) curves $\theta - \lg U_g$ are found to be straight lines, save the case of very lowest potentials ($U_g = -5$ V); 2) both straight lines, corresponding to $i = 150$ mA and $i = 225$ mA, run in parallel.

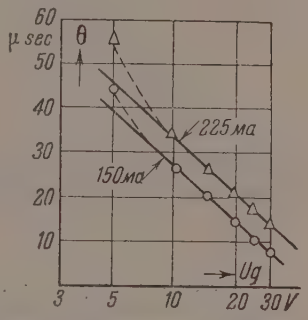

Fig. 3. Comparison of experimental data by Druskina and Krassilov with formula (5).

As regards the deviations observed with small values of U_g, we have already seen above a number of reasons, on the ground of which we can anticipate them precisely with small values of U_g: the electronic component of current; initial velocity of ions; deviation from the exponential course of deionization; difference in potentials of cathode and plasm—all these are circumstances which we neglected in drawing conclusion (5). Due account taken of all these factors would, however, have strongly complicated the calculation and could hardly have been made use of in practice.

By way of summing up, we may say that formulae (5) and (5′) resulting from elementary considerations of theory, as well as from firmly established evidential facts, express very well the dependence of the time of restoration of controllability of discharge upon the value of the discharge current, the rate of the deionization process and the controlling tension, provided the latter is not too small.

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CHEMIE

DIE ABHÄNGIGKEIT DER VISKOSITÄT DER HOCHMOLEKULÄREN FETTSÄUREN VON DER TEMPERATUR UND DEM GRADE IHRER UNGESÄTTIGTHEIT

Von G. B. RAWITSCH

(Vorgelegt von N. S. Kurnakov, Mitglied der Akademie, d. 23. IX. 1938)

In der vorliegenden Arbeit stellten wir uns das Ziel, eine Universalgleichung für die höheren Fettsäuren mit 18 Kohlenstoffatomen zu finden, in der die absolute Viskosität η als Funktion der Temperatur t und der nach der Methode von Hüble bestimmten Jodzahl J ausgedrückt wäre:

$$\eta_t = F(J, t).$$

Hierbei mussten offenbar 2 Probleme gelöst werden.

Erstens in mathematischer Form den Einfluss der durch ihre Jodzahl charakterisierten doppelten Bindungen im Molekül auszudrücken; zweitens. einen den Einfluss der Temperatur auf die Viskosität der Fettsäuren bei der gegebenen Jodzahl J charakterisierenden Ausdruck zu finden.

Die Werte der Viskosität in Zentipoisen η und der Jodzahlen J wurden von uns experimentell gefunden. Die Methodik ist in unseren früheren Arbeiten [1] beschrieben. Es wurden Präparate der Firma Schering-Kahlbaum benutzt. Unsere Ergebnisse im Vergleich mit den Daten einiger anderer Forscher sind in der Tabelle 1 angegeben.

Auf Grund der Wiederholbarkeit der Resultate, sowie der guten Übereinstimmung mit den Daten anderer Forscher, kann man die angeführten experimentellen Werte als genügend zuverlässig betrachten.

Bei der Entscheidung der bezeichneten Fragen liessen wir uns von folgenden Erwägungen leiten. Wie aus der Abb. 1 zu ersehen ist, bilden im Koordinatensystem η—J (Viskosität in Zentipoisen—Jodzahlen) die Viskositätsisothermen für die höheren Fettsäuren mit verschiedenen Jodzahlen Gerade, wobei die Viskosität mit der Erhöhung des durch die Jodzahl ausgedrückten Ungesättigtheitsgrades systematisch fällt. Umgekehrt steigt die Viskosität mit jedem Zerreissen der doppelten Bindung unter Anschluss von 2 Atomen Wasserstoff (was dem Übergang von einer weniger gesättigten zur gesättigteren Säure entspricht) systematisch an. Den zweifellosen Zusammenhang der Erhöhung der Viskosität mit der Struktur der Säuren kann man auch röntgenographisch verfolgen, indem man den Durchmesser der Ringe der Debyegramme mit dem Wert der Jodzahlen vergleicht.

Auf Grund des Gesagten kann man für den isothermischen Fall (fast für alle Isothermen) die erste Gleichung

$$\eta_t = \eta_t^c - K_t J \qquad (1)$$

Tabelle 1

Viskosität der höheren Fettsäuren C_{18} mit verschiedener Jodzahl (in Zentipoisen)*

Bezeichnung der Fettsäure C_{18}	Jodzahl nach Hüble	Temperatur in °C										
		20	30	40	50	60	70	80	90	100	110	120
Linolensäure . . .	256.6	18.0	13.0	10.3	7.9	6.3	5.2	4.4	3.7	3.0	2.6	—
Gemisch von Linolen- und Linolsäure	232.6	—	14.2	10.9	8.8	7.2	6.1	—	—	—	—	—
Linolsäure	200.1	23.4	16.6	12.3	9.8	7.5	6.1	5.2	4.3	3.6	3.0	—
Oleinsäure	89.7	39.1	25	18.0	13.6	10.3	8.0	6.3	5.2	4.3	3.4	2.8
			25.6(2)									
Gemisch von Olein- und Stearinsäure	42.9	—	—	—	—	11.7	9.4	7.1	5.6	4.7	—	—
Stearinsäure . . .	—	—	—	—	—	—	—	8.30	5.66	4.88	4.18	3.37
							9.39(4)	—	5.63(4)	—	—	—
							11.6(3)	—	7.95(3)	5.12(3)	—	3.37(3)

vorschlagen. Hier ist K_t—der Temperaturkoeffizient der Isothermen, η_t^c—die Viskosität der Stearinsäure mit der Jodzahl $J=0$ bei der Temperatur t (in gewöhnlichem oder unterkühltem Zustande, wobei die auf der Abb. 1 mit doppelten Kreisen bezeichneten Punkte durch Extrapolation der Isothermen bis zum Schnittpunkte der der Viskosität der Stearinsäure entsprechenden Ordinate gewonnen wurden).

Aus der Gleichung (1) ist zu ersehen, dass die Frage der Temperaturabhängigkeit der Viskosität der Stearinsäure (η_t^c) entschieden und eine die Veränderung des Winkelkoeffizienten der Isothermen K_t mit der Temperatur charakterisierende Gleichung gefunden werden muss; von der Behandlung der Untersuchungsobjekte, als assoziierte Flüssigkeiten, die ihren Assoziationsgrad mit der Temperatur verändern, ausgehend benutzten wir die Exponentialgleichung von Fulcher-Tammann (5):

$$\eta_t^c = \eta_0 \cdot 10^{\frac{c}{t-t_0}}. \qquad (2)$$

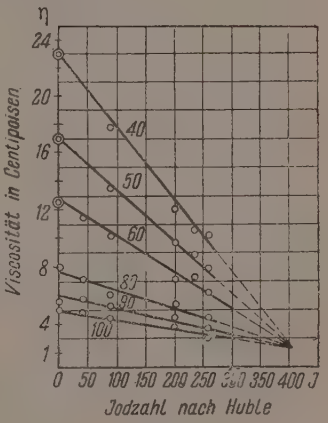

Fig. 1.

Hier sind η_0, c und t_0 gewisse, die gegebene Flüssigkeit charakterisierende Konstanten.

Die dritte Gleichung von analogen Typus

$$K_t = K_0 \cdot 10^{\frac{c'}{t-t_0'}} \qquad (3)$$

wurde zur Auffindung der Werte bei verschiedenen Temperaturen benutzt.

* Die der Literatur entnommenen Viskositätswerte der Olein- und Stearinsäure sind in der Tabelle mit den entsprechenden Hinweisen auf die Literatur angegeben.

Indem wir die Gleichung (1) zusammen mit den Gleichungen (2) und (3) lösen, erhalten wir folgende Gleichung (4):

$$\eta_t = \eta_0 \cdot 10^{\frac{c}{t-t_0}} - K_0 \cdot 10^{\frac{c'}{t-t'_0}}. \qquad (4)$$

Im Nachstehenden sind die Tabellen 2 und 3 angeführt, die die Genauigkeit der vorgeschlagenen Gleichung charakterisieren, sowie die Werte der in der Formel enthaltenen, nach 3 Werten der Viskosität und der Winkelkoeffizienten bei 3 Temperaturen berechneten Konstanten angegeben (die Methode der Berechnung der Konstanten ist in einer ganzen Reihe von Arbeiten dargelegt).

Tabelle 2

Werte der Konstanten der vorgeschlagenen Gleichung

$\lg \eta_0$	c	t_0	$\lg K_0$	c'	t'_0
$\overline{2}.13$	750	—192	$\overline{1}0.59$	4 063.5	—460

In der Tabelle 3 sind in den Kolonnen I die experimentellen Viskositätswerte, in den Kolonnen II die nach der vorgeschlagenen Gleichung (4) berechneten Werte gegeben.

Tabelle 3

Kongruenz der experimentellen und berechneten Viskositätswerte in Zentipoisen

Bezeichnung der Fettsäure C_{18}	Temperatur in °C													
	30		40		50		60		90		100		200	
	I	II	I	II	I	II	I	II	I	II	I	II	I	II
Linolensäure	13.0	12.7	10.3	9.6	7.8	7.8	6.3	6.4	3.7	3.6	3.0	3.2	—	—
Linolsäure	16.6	16.9	12.3	12.5	9.8	9.8	7.5	7.8	4.3	4.3	3.6	3,6	—	—
Oleinsäure	25.1	25.3	18.0	18.2	13.6	13.8	10.3	10.6	5.2	5.3	4.3	4.4	—	—
Stearinsäure	—	—	—	—	—	—	—	—	5.7	6.1	4.9	5.0	1.1[3]	1.1

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CHEMISTRY

ELECTRONIC DIFFRACTION IN THE SURFACE LAYERS OF METAL ALLOYS

By V. KASSATOCHKIN

(Communicated by V. A. Kistiakowsky, Member of the Academy, 31. X. 1938)

For obtaining a clear-cut interference picture in the reflection of cathode rays from the surface of a solid specimen of metal, it is necessary to carry out preliminarily a thorough preparation of the surface. The most simple and convenient way of preparing a metal surface is polishing it with emery paper of a suitable No. According to the ideas of the majority of investigators working in this field, the advantage of such treatment lies in the formation of a sufficiently large number of small projections on the surface of the specimen which are apt to be well transilluminated by the electronic beam sliding over the surface of the specimen([1]). However, the process of polishing or any other way of mechanical treatment of a metal surface (e. g. cutting. etc.) produces plastic deformation in a certain thickness of the outer layer of the specimen, along with a temporary rise in temperature. In the treatment of alloys it is moreover natural to expect deep physico-chemical processes taking place in the surface layers of the specimen, these processes being brought about by the well-known influence of strain upon the rate with which an alloy approaches the state of thermodynamic stability. Various authors carrying out their investigations with quite a number of X-ray methods have succeeded in ascertaining the dependence of the depth of the deformed layers and the character of structural changes upon the method of treatment of the surface and, in certain cases, they have found far reaching processes of phase transformations([2]).

The picture of electronic interference reflects the structure of only a very thin surface layer substantially changed under the effect of deformation. and, consequently, it fails to characterize the condition of the alloy throughout the bulk of the specimen.

In the present investigation the picture of electronic interference obtained from different alloys has been compared with the interference picture of X-rays for the same materials. The comparatively great depth of penetration of X-rays into the substance permits to obtain an idea of the structure of the deeper layers of the bark of the alloy, which presumably comes very near to the structure of the whole volume of the specimen.

Our experimental procedure was as follows. From specimens of three different alloys, whose surface had preliminarily been polished with emery paper, electronograms were taken in electrographs by the reflection method. The electrograph had a hot cathode, and the distance from the experimental material to the photoplate was 245 mm. Reflection exographs of the same material were taken in Debye cells of the ordinary radius with non-filtered radiation of iron. The following results were obtained.

Alloy 1. Steel of the following composition: C—0.77%, Mn—0.2%. S—0.11%. Quenched in water from 800°. Tempering 30′ at 150°.

The steel exograph contains intense lines of martensite (the α-phase with body-centered lattice) and less sharply marked lines of austenite (the γ-phase with face-centered cubic lattice). Netchvolodov[3] has developed a method for the determination of the amount of residual austenite in steels, based on a visual comparison of the intensities of the different lines of both phases. By using this method, it was found possible to evaluate the content of the γ-phase in the test sample, which came to about 20%.

The electronogram of this alloy contains only the α-phase lines and is perfectly identical to the electronogram of pure iron. The difference between the two was found to lie only in a more pronounced background of uncoherently dispersed electrons in the case of the steel photograph. No traces of lines corresponding to the γ-phase could be detected in the electronogram of steel.

Alloy 2. Steel of the following composition: Cr—12%; Mn—8%; Ni—4%. Quenched in water from 1100°.

The exograph of the ground specimen of the alloy displays lines of the α-phase and the γ-phase. Jugded on the basis of the equal intensity of line (111) for the γ-phase and line (110) for the α-phase, the content of austenite in the sample could be evaluated as amounting to about 60%. The electronogram of this alloy, characterized by sufficiently sharply marked interference lines, fails just as in the aforesaid case to contain any traces of lines corresponding to the γ-phase. In this instance the size of the background of uncoherently dispersed electrons increases still further.

Alloy 3. Brass, containing 38% Zn. Quenching from 830°.

The exograph of a ground sample of the alloy exhibits the presence of the α-phase (face-centered cubic lattice) and of the β-phase (body-centered cubic lattice), available in approximately equal ratios.

The electronogram of this alloy displays only lines of the α-phase. In comparing the electronograms of pure copper and the alloy in question, a difference is to be found only in the appearance of a much larger background in the photograph of brass. The insufficient degree of precision of the experiment prevented us from noticing any changes in the constants of the lattice.

A very characteristic feature, common to the electronograms of all the three alloys investigated, is the fact of the disappearance of the lines of those phases which are unstable at low temperatures.

The most plausible explanation of the fact of the disappearance of the lines of the β-phase in the electronograms of brass and of the lines of the γ-phase in the plates of steel lies, as we believe, in the process of disintegration of these phases in a thin surface layer, taking place under the influence of intense deformation and a certain rise in temperature.

The effect of deep changes in phase composition in thin surface layers is undoubtedly of frequent occurrence in connexion with various ways of mechanical treatment of the surface of alloys (e. g. grinding, burnishing, cutting, drilling, etc.). The altered layer is, however, in many cases so very thin that the method of X-ray diffraction appears to be too rough for its detection. Electronic diffraction, as a method of investigation for the case in hand, appears to have a number of outstanding advantages.

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ENGINEERING GEOLOGY

FRICTION AND COHESION OF COHERENT ROCKS

By F. P. SAVARENSKY and N. V. KOLOMENSKY

(Communicated by A. D. Arkhangelsky, Member of the Academy, 11. XI. 1938)

The available methods of determination of the friction and cohesion values of coherent rocks cannot be considered quite satisfactory in so far as the variable of internal forces is not taken into account. Owing to this, the methods and formulae for calculating the stability of natural and artificial banks, retaining walls etc., logical as they are in themselves, fail to yield satisfactory results, inasmuch as the friction and cohesion values made use of are erroneous. It proved therefore so far necessary to ground such calculations chiefly either on experience and intuition, or on arbitrarily assumed safety margins, which enhances the cost of construction without protecting it against deformation.

According to Culomb's law for friable matter, a direct dependence between the shearing force value and the normal load should be assumed within the range of common loads, while in coherent rocks without a rigid connexion between the grains, say, in clays, the shearing forces are made up of the strain necessary to overcome friction and the internal forces of cohesion, the value of which depends on the moisture of the ground or on the degree of its density, being no constant, as it is still assumed in different manuals and text-books.

The work, carried out by the Engineering Geology Department of the Geological Institute of the Academy of Sciences, enables us to draw the following conclusions:

a) The mechanism of both the friction and the cohesion phenomena in coherent grounds is the same and consists in cleaving off of aqueous films enveloping the ground particles both in the presence of an external vertical load and without it.

Experimental work and theoretical conceptions based on the so-called film-theory make us believe that friction and cohesion obey laws which are quite different from those these phenomena are accounted for at present.

b) The actual cohesion value depends on the relative moisture and the structural preservation of the ground. The relative moisture is taken equal to the relation of the moisture shown by the ground in shearing, to the upper limit of plasticity of the given ground (according to Atterberg).

According to data derived from 66 determinations made for different argillaceous grounds, the dependence between the cohesion and the relative moisture fulfills (at a certain approximation) the cubic parabola equation (Fig. 1). Within the range of moisture investigated the curve shown in Fig. 1

tends to intersect the abscissae axis at a point corresponding to the relative moisture unit. This corresponds to the theoretical conception of the action of the ground in shearing, namely that with a moisture corresponding to the upper plasticity limit the ground passes to a fluid condition, losing its cohesion.

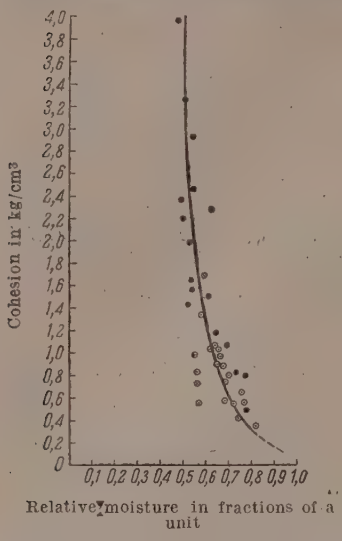

Fig. 1. Curve of dependence of cohesion on relative moisture (undisturbed structure). ●—Jurrassic clay; ⊙—Quaternary loams.

As may be seen from Graph 1, the curve which shows the dependence of cohesion on the relative moisture, above the relative moisture range 0.5—0.75, may be taken for a straight line, corresponding to condensing loads of about 1—6 kg/cm². The considerable cohesion values, obtained in the course of the experiments, are confirmed by breaking tests.

The actual friction coefficient depends upon the moisture of the ground, decreasing with the latter; however, within the range of small vertical loads (1—6 kg/cm²) it may be regarded as a constant value.

This is confirmed by shearing tests made after the method of Savarensky(1) with Jurassic clays and Quaternary loams; on samples of both a disturbed and undisturbed structure. This method includes two procedures: first, the usual shear of the ground under various increasing loads, and secondly, the shear of the ground after its condensation under the same loads taken off before shearing. The difference between two shearing forces with reference to the vertical load corresponds to the friction coefficient, while the shearing force without the action of a vertical load corresponds to the cohesion of the ground (Fig. 2).

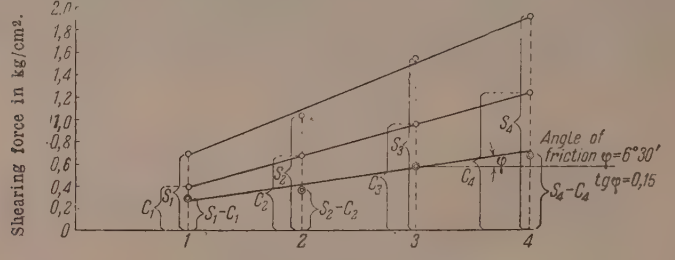

Fig. 2. Shift-diagram after Savarensky (ground mass). Specimen № 12 (Jurassic clay). Friction coefficient in kg/cm²

Thus, within the range of small specific loads (from 1 to 6 kg/cm²) the friction coefficient may be regarded as increasing proportionally to the external vertical load. The friction coefficient f is determined according to the formula:

$$f = \frac{S_n - C_n}{N_n},$$

where S_n—the shearing force at a load N_n; C_n—the shearing force without a load, i. e. a cohesion value corresponding to the given condensing load; N_n—the vertical load.

Besides, the influence of repeated shearings on the friction and cohesion values of coherent grounds has been established. In case of repeated shears, the following two factors are operative, which act in the same direction:

a) the «shearing habit» of the ground—the formation of furrows and reorientation of particles on the surface of the shear;

b) the decrease of the shearing force at the expense of elastic deformation of the film, while the load is taken off the specimen.

The factors established may serve to account for the phenomena that take place in the case of shiftings of the ground masses of moving landslips.

The investigation of ground friction and cohesion should proceed by way of studying both the solid and the liquid phase of the ground and their behaviour in shifts, as affected by various factors, among which rock structure and physico-chemical phenomena seem to play a considerable part. The investigation should also proceed by way of studying natural banks of rocks of different genetical types, with the view to find out the applicability of theoretical data to the evaluation of the stability of these rocks.

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MINERALOGY

ON THE ORIENTATION OF QUARTZ CRYSTALS IN THE VEINS OF THE ALPINE TYPE IN THE SUB-ARCTIC URALS

By G. G. LAEMMLEIN

(Communicated by W. I. Vernadsky, Member of the Academy, 27. XI. 1938)

The crystals of quartz (rock crystals) in the veins of the Alpian type, in quartz-chloritic, quartz-sericitic schists of the Sub-Arctic Urals usually have their axis of the third order *c* inclined towards the wall of the fissure, and but very rarely perpendicular to the latter. As a characteristic feature of some veins, of rather frequent occurrence are crystals whose axis of the second order *a* is perpendicular to the wall, the positive end of the axis being directed forwards (3).

The cavities, in which the growth of rock crystals proceeded, are due to tension fissures cutting quartz-sericite-chloritic schists almost perpendicularly to the schistosity. Naturally enough, the growth of rock crystals started at places, where the grains of quartz had been exposed on the fissure walls. Quartz as a rock forming mineral is concentrated in the schist in the form of small, up to 2—3 mm thick (sometimes thicker) finely-grained, lens-like, pinching-out veinlets parallel to the schistosity. The quartz grains of such lenses broken by fissures served as germs for the further growth of rock crystals. Thin-sections, cut perpendicularly to the fissure wall with a druse of crystals grown on it, show under the microscope a sharp boundary between the rock and the small lenses of quartz, cut by the fissure. The grains of quartz exposed by the fissure which have almost isometric contours, as throughout the rock, grow in the cavity and assume an elongated form, some of them developing into big crystals. E. Schnaebele (8) and I. Holzner (2) point to the fact that quartz crystals in the fissures of schists are nothing else but the continuation of rock grains, and that their axes *c* are often parallel to the fissure wall or perpendicular to the schistosity.

Since the growth of rock crystals begins on the germs exposed by a fissure, the orientation of crystals in such a vein is predetermined by that of the quartz grains in the rock.

As early as in 1906, G. B. Trener (9) pointed to the fact that quartz grains in schists are situated their axis *c* being perpendicular to the schistosity. W. Schmidt (7) and Br. Sander (5), using Fedoroff's stage to determine the orientation of quartz grains in thin-sections, showed the existence of various regularities accounting for the position of quartz grains in a rock which had undergone tectonic influence. These laws may be expressed by statistical graphs of various types. A series of such typical graphs (4) shows the pole of the plane of tension fissure to coincide with the minimum outcomes of the optical axes of quartz.

The author succeeded in measuring the orientation of 500 quartz grains in a thin-section of a quartz-sericite-chloritic schist (Sura-iz mountain, Sub-Arctic Urals) cut parallel to the tension fissure.

Fig. 1 shows a statistical graph of the schist in Schmidt's projection. The centre of the graph coincides with the pole of the break fissures.

A thick line in the middle encircles a field containing no outcomes of optical axes of quartz at all. The highest ratio of quartz axes falls within a broken tension belt which coincides with the fissure plane. The vertical line represents the trace of schistosity perpendicular to the fissure plane.

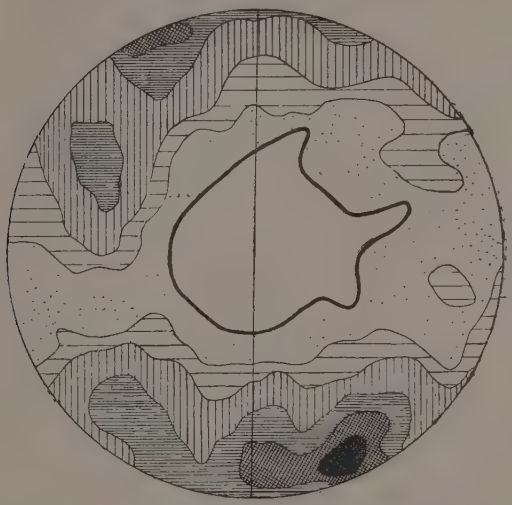

Fig. 1. Quartz-sericite-chloritic schists, Sura-iz, Sub-Arctic Urals. Graph of orientation— 500 grains of quartz. The plain of projection coincides with the fissure of the break. 0—½—1—2—3—4 —5%.

The thin-section perpendicular both to the schistosity and to the tension fissure displays the same type of maxima and minima distribution. Without dwelling upon the details of the graph itself, I shall but emphasize some conclusions, which may be made use of for the study of genesis of rock crystal deposits.

In the case of a random distribution of a great number of quartz germs on the wall of a fissure, this distribution being due to geometrical selection that took place during the process of growth ([6],[1]), there would develop crystals stretching along the *c* axis, closely adjacent to each other and covering the wall of the fissure by a continuous crust. This must be due to the fact that among a great number of grains, possessing a random orientation, there would always exist a sufficient number of such having their axis *c* perpendicular to the fissure wall. This position being most favourable for the growth of quartz crystals, only those oriented according to the above-mentioned law of geometrical selection would have chance to develop into large individuals.

But when the germs on the surface of the tension fissure, where the growth of rock crystals begins, have a previously set up orientation shown in the graph, then there are found no germs having a position convenient for growth, i. e. having the axis *c* perpendicular to the wall. On the other hand, there is a sufficient number of germs with the axis *c* parallel to the plane of fissures.

Only very few among those germs have the axis *a* directed conveniently for growth, i. e. perpendicular to the wall, and with the positive end directed forward, so that they are able to grow into big flat crystals. In some deposits these crystals grow in the form of twisted ones (3). In a small number of germs the axes *c* are inclined towards the plane of fissure. They give rise to the biggest crystals in the vein.

The more scattered are the germs, the orientation of which is convenient for growth, the bigger are the crystals finally grown out of them.

The orientation of quartz grains in the rock, giving on the graph for the quartz axes a clean-cut broad minimum coinciding with the pole of tension fissures, seems to be one of the conditions for the formation of isolated, big quartz crystals in the veins of the Alpine type and of the appearance in these veins of flat quartz crystals stretched along the axis *a*.

The orientation of quartz may be seen with particular distinctness at the initial stage of the growth of crystals on the fissure wall, while the latter is not yet covered by a continuous crust of crystals. Later on, as the crust of quartz gets thicker, it often separates from the rock, especially from the hanging wall, and may either continue to grow overhanging the cavity of the fissure, or fall on the lower wall of the latter. In this case the druses may be crushed into separate crystals, which will continue to grow lying on the tops of crystals belonging to the lying wall. Owing to the small tectonic shifts following the crystallization of the vein, in the veins of Alpine type in the Sub-Arctic Urals almost no veins with cavities, rock crystal cellars or caves are observed. They are destroyed, closed, filled with crushed schists. Only in exceptional cases the crystals of quartz occur in the same position as they grew in the vein; usually they are separated from each other and mixed with rock fragments, chloritic sand, all the parts being cemented by the ice of perpetual frost.

Thus, we are actually able to judge of the initial orientation of large quartz crystals on the fissure walls by the orientation of small crystals grown in the fine jointing fissures parallel to the rock crystal bearing vein.

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PHYSIOLOGIE VÉGÉTALE

L'ÉTUDE ANATOMIQUE DE LA PROPAGATION DES GYPHES DES CHAMPIGNONS DANS LE BOIS MORT DE L'ÉPICÉA DU CAUCASE (*PICEA ORIENTALIS* CARR.)

Par ANDRÉ A. YATSENKO-KHMÉLEVSKY

(Présenté par A. A. Richter, de l'Académie, 14. XI. 1938)

La propagation des gyphes des champignons dans le bois dépend de sa structure anatomique. Il est clair, par exemple, que la croissance des gyphes le long des éléments prosenchymateux rencontre moins d'obstacles que dans la direction radiale, car les parois des fibres occasionnent une résistance importante à la propagation du mycélium. Néanmoins, l'analyse microscopique du bois infecté ne peut fournir que des preuves indirectes à cette assertation (l'orientation longitudinale des gyphes par rapport à l'axe du tronc). Au contraire, dans bien des cas, lorsqu'il s'agit du bois vivant, on trouve même des indications sur la localisation des gyphes dans les rayons médullaires [1], ce qui prouve qu'il faut distinguer la propagation des gyphes dans le bois mort de celle du bois vivant. Dans le bois mort il n'y a que la structure anatomique qui influe sur le caractère de la propagation, tandis que dans le bois vivant, surtout dans l'aubier, la croissance des champignons dans les cavités des éléments prosenchymateux ne peut avoir place, car elles sont occupées par la sève ascendante. Ceci explique la tendance des gyphes de se propager par les rayons médullaires, seuls éléments du bois vivant contenant de l'air.

Pour pouvoir analyser la propagation des gyphes des champignons dans le bois mort, il fallut organiser l'inoculation du bois par des procédés spéciaux. Le but de notre travail a été d'obtenir la pénétration du champignon dans le bois par une des trois sections: transversale, radiale et tangentielle. À ce but nous avons préparé du bois de l'épicéa des échantillons de dimensions de $2 \times 2 \times 2$ cm. Le bois des échantillons possédait 15% d'humidité. Les échantillons, après le polissage, étaient enveloppés dans l'étain battu, sans employer de colle, pour éviter sa pénétration dans le bois. Les échantillons enveloppés étaient couverts de laque d'asphalte et ensuite plongés dans la paraffine fondue. Une protection si complexe était nécessaire pour garantir le bois de la pénétration des champignons dans les endroits imprévus. Une des surfaces des échantillons restait libre, pour faciliter la pénétration de l'air dans le bois. Dans la surface opposée à celle restée ouverte on faisait une ouverture avec un burin, 3 mm de diamètre, pour la pénétration du champignon. Les bords de cette ouverture étaient enduits de la laque d'asphalte (photo). En tout nous avons préparé 39 échantillons, dont

13 avaient l'ouverture sur la surface transversale, 13 sur la surface radiale et 13 sur la surface tangentielle.

Pour notre expérience nous avons choisi deux champignons: *Fomes igniarius* et *Poria vaporaria*. On sait que les destructions occasionnées par *Fomes igniarius* se rapportent au type «corrosif» ([2]), tandis que *Poria vaporaria* provoque la destruction du type «destructif»([3]). Comme substratum pour les champignons nous avons employé le moût de bière avec 2% d'agar-agar, dans des bocaux de 200 cm³. L'inoculation du substratun a été faite le 26. IV. 1937 avec *Fomes igniarius* et le 19. V. 1937 avec *Poria vaporaria*. Quand le mycélium des champignons a complètement couvert le substratum, les échantillons ont été placés dans les bocaux, l'ouverture sur le mycélium et la surface libre en haut.

Dans des délais déterminés, les échantillons étaient tirés des bocaux, débarrassés de l'étain battu et immergés dans l'alcool. Pour avoir un tableau complet de la propagation du champignon dans l'intérieur de l'échan-

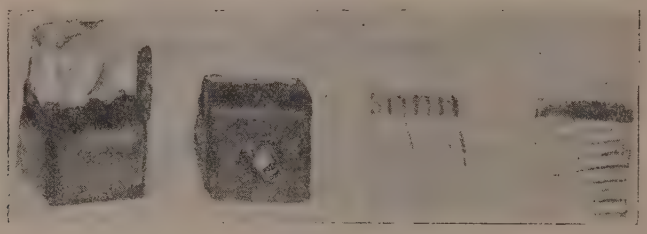

Fig. 1.

tillon, ces derniers étaient divisés en 4 parties égales. Une ou deux des prismes obtenues étaient analysées. On prenait des coupes pour l'étude microscopique en douze endroits de chaque prisme, dans six endroits de chaque surface intérieure. Les coupes ont été teintes par l'aniline bleue, selon la méthode proposée par le prof. Kurssanov ([4]), mais simplifiée vu la grande quantité des préparations—de chaque échantillon on prenait plus de 100 coupes. Ainsi, les coupes n'étajent pas chauffées dans la solution d'aniline bleue et le lavage était fait dans l'eau. Il est nécessaire d'ajouter que la méthode simplifiée donne néanmoins d'excellents résultats.

Les échantillons infectés par *Fomes igniarius* ont été analysés 12 et 35 jours après le commencement de l'expérience. L'analyse a montré que: 1) la vitesse de la propagation des gyphes le long des trachéides surpasse de 8—10 fois la vitesse de la propagation à travers les trachéides. En 35 jours, les gyphes se sont propagés le long des trachéides à une distance de 2 cm. 2) La propagation des gyphes à travers les trachéides dans la direction radiale surpasse de 1.5 fois la propagation en direction tangentielle, car dans le premier cas les gyphes croissent le long des rayons médullaires, d'où elles sortent dans les cavités des trachéides. 3) Le passage des gyphes d'une trachéide à l'autre se fait par les ponctuations aréolées, ou, ce qui est plus commun, par des petits canaux que les gyphes creusent dans les parois cellulaires. 4) Après 35 jours de l'expérience nous n'avons pas pu constater des indices de destruction des membranes cellulaires, à l'exception de petits canaux servant de passage aux gyphes. 5) La propagation longitudinale du champignon dans le bois initial est plus intense que dans le bois final; le bois final offre aussi une résistance sensible à la propagation des gyphes à travers les trachéides dans la direction radiale.

Les échantillons infectés par *Poria vaporaria* ont été examinés 7 et 17 jours après l'inoculation. L'analyse microscopique a montré que: 1) la vitesse de la propagation des gyphes le long des trachéides dans les échantillons à l'ouverture sur la surface transversale surpasse plus de dix fois la vitesse de la propagation à travers les trachéides dans les échantillons avec les ouvertures sur les surfaces transversale, radiale et tangentielle. Les gyphes se sont propagées, durant 17 jours, à une distance de 0.5 cm. La propagation longitudinale des gyphes dans les échantillons à l'ouverture sur la surface radiale et tangentielle ne surpasse que très peu la propagation à travers les trachéides chez les mêmes échantillons. 2) La propagation des gyphes à travers les trachéides dans la direction radiale ne diffère sensiblement de la propagation dans la direction tangentielle. 3) Le passage des gyphes d'une trachéide à l'autre s'effectue, à ce qu'il paraît, seulement par les ponctuations aréolées. 4) Durant 17 jours de l'expérience, dans les portions du bois où les gyphes sont nombreuses, une destruction importante des membranes cellulaires a été notée: l'amincement des parois des trachéides, l'apparition des fissures et des déchirements de la membrane secondaire. 5) La propagation des gyphes est plus intense dans le bois initial que dans le bois final.

Les mesures ordinaires de protection du bois contre la moisissure—l'essuyage avec l'alcool et le grillage—n'ont pas pu être appliquées à nos échantillons couverts de la laque d'asphalte et de la paraffine. C'est pourquoi nous avons constaté l'apparition des champignons filamenteux (*Penicilium* sp.) sur les surfaces libres des échantillons et même partiellement sur les surfaces couvertes latérales. Les gyphes de ces champignons ne pénètrent dans le bois qu'à des distances très insignifiantes (quelques mm), dans les endroits que les gyphes de *Poria* et de *Fomes* n'ont pas atteints. Chaque fois que la pénétration des champignons filamenteux a été notée, le caractère de sa propagation était analysé minutieusement, ce qui a permis d'établir les particularités qui la distinguent de la propagation de *Poria* et de *Fomes*.

Les gyphes des champignons filamenteux, qui pénètrent dans le bois par la surface transversale, se propagent longitudinalement par les trachéides, en passant d'une trachéide à l'autre par les ponctuations aréolées. Leur propagation est peu importante et elles n'atteignent que très rarement une distance de 3—4 mm. Les gyphes qui croissent par la surface radiale, ou, surtout, par la surface tangentielle, comme règle, se propagent premièrement par les rayons médullaires, à une distance de quelques mm (parfois jusqu'à un cm). Très souvent elles sortent par des ponctuations aréolées dans les cavités des trachéides et se propagent dans la direction longitudinale. Il est nécessaire de noter que dans les rayons médullaires les gyphes se propagent dans une assise de cellules, évitant les assises supérieures ou inférieures. La propagation longitudinale des gyphes des *Penicilium* a été notée presque exclusivement dans le bois initial. Aucun indice de la destruction du bois, comme il fallait s'y attendre, n'a pas été noté.

Résumons les conclusions principales de notre travail.

1. Le caractère de la propagation des gyphes dans le bois est différent chez les champignons différents. Les gyphes des champignons filamenteux (*Penicilium* sp.) se propagent surtout par des rayons médullaires, tandis que les gyphes des champignons destructeurs du bois étudiés (*Fomes igniarius* et *Poria vaporaria*) se propagent surtout le long des trachéides. La croissance dans cette direction s'effectue à peu près 10 fois plus vite qu'à travers les trachéides.

2. Entre la propagation de *Fomes igniarius* et *Poria vaporaria* on peut noter des différences, liées probablement avec la physiologie de ces deux champignons. Les gyphes de *Fomes igniarius* possèdent la capa-

cité de creuser avec une facilité comparative des petits canaux dans les parois des trachéides, par lesquels elles pénètrent d'une trachéide à l'autre. À ce qu'il paraît, les gyphes de *Poria* sont privées de cette capacité et sont forcées de passer d'une trachéide à l'autre par les ponctuations aréolées, ce qui fait que, dans nos expériences, la vitesse de la propagation des gyphes de *Fomes igniarius* surpasse celle de la propagation de *Poria vaporaria*. Néanmoins, les gyphes de ce dernier, même dans les premiers stades de leur introduction dans le bois, provoquent dans celui-ci des destructions considérables, comme l'amincissement des membranes des trachéides, l'apparition des fissures et les déchirements de la membrane secondaire. *Fomes igniarius*, au contraire, ne provoque pas de destructions dans le bois, à l'exception d'étroits canaux que les gyphes creusent dans le bois dans leur passage d'une cellule à l'autre. Ainsi, la différence entre *Poria vaporaria* et *Fomes igniarius* consiste en ce que chez *Poria* la destruction du bois accompagne la propagation des gyphes, tandis que chez *Fomes igniarius* la destruction n'apparaît que longtemps après la propagation des gyphes dans le bois.

3. La propagation des deux champignons étudiés (surtout *Poria vaporaria*) est plus intense dans le bois initial que dans le bois final, ce qui peut être expliqué par les parois plus épaisses des trachéides du bois final qui entravent le passage des gyphes.

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ZOOLOGY

THE BODY TEMPERATURE OF REPTILES IN NATURAL SURROUNDINGS

By A. SERGEYEV

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The present communication is an exposition of certain materials, which have been gathered in support of the statement made by A. V. Ryumin regarding potential homoiothermism of reptiles during the period of their activity (the active part of the 24-hour cycle). The material has been collected in the Turkmen Socialist Soviet Republic, in the sandy country west of the town Merva, during the month of May, 1938. In view of the fact that microclimatic conditions in habitats of reptiles in general [with the exception of data furnished by Kashkarov(1); also see Mosauer(2)] are totally unknown, temperature measurements have been taken in different parts making up the habitat of the species concerned, and the records offer a key to the understanding of the 24-hour cycle.

The body temperature of the animals has been measured by means of an Assmann thermometer (in lizards and snakes in the throat, in tortoises—per os), whereby temperature was recorded only in specimens caught «at unawares», because there is a sharp rise of temperature in reptiles after racing over sun-heated ground (in agama, for instance, from 33° to 38° within 5—6 minutes), and such a record would not give the idea of «natural» temperature. Temperature measurements have been taken of the following diurnal species: reticulate scapteira (*Eremias grammica*), auriculate round-headed lizard (*Phrynocephalus mystaceus*), round-headed lizard Radde (*Phr. raddei*), steppe-agama (*Agama sanguinolenta*) and tortoise (*Testudo horsfieldi*). More or less adequate material is available but on the last three species.

Generally speaking, our data are in full agreement with the above-mentioned assertion of A. V. Ryumin. The temperature in all the enumerated reptilians during the period of activity was above 30°, namely:

1.	*Phrynocephalus mystaceus*	from + 34	to + 35°
2.	» *raddei*	» + 36	» + 37°
3.	*Agama sanguinolenta*	» + 36	» + 37°
4.	*Eremias grammica*	» + 34	» + 36°
5.	*Testudo horsfieldi*	» + 32	» + 36°

Note. Over 20 measurements have been taken of each of the species.

The quoted figures present but a general picture of the range of body temperature in reptiles during the active period of the 24-hour cycle. As

far as the tortoise, agama and round-headed lizard Radde are concerned, the material on hand enables one to judge about changes in body temperature maintained by these species during the 24-hour period.

In the graph dotted lines represent temperatures in tortoises recorded under natural conditions during the 24-hour period on May 24, 1938 (a number of individual recordings on other days show similar figures). For the sake of a clearer understanding of the graph, we are giving below a brief account of the 24-hour cycle of the tortoise (after field observations of single tortoises and numerical account): 6.05 a. m.—sunrise, 6.42 a. m.—waking, from 6.42 to 8.30 a. m.—warming in the sun at the entrance of the burrow, from 8.30 to 11.30 a. m.—feeding, from 11.30 a. m. to 4.50 p. m.—resting in the holes or under dense bushes, from 4.50 to 6.50 p. m.—feeding, about 6.50 p. m.—going to sleep in the burrows, from 7.25 p. m. to 6.42 a. m.—night sleep.

A juxtaposition of the graph and the 24-hour cycle shows a wide range of body temperature in the tortoise within the 24-hour period, namely: 1) at night the temperature of tortoises is sustained at 23—24°; 2) after exit from the burrow and insolation, the temperature rises within short time up to 30—32°, 3) during the active period temperature is maintained in the neighbourhood of 32—34°, reaching as high as 36°, and 4) during the hotter part of the day the tortoise is taking refuge either in the burrow or under a shrub.

It is noteworthy, that in burrows inhabited by tortoises, at a depth of 50—70 cm, temperature fluctuations, which are rather sharp on the surface, are greatly attenuated. Table 1 presents the 24-hour temperature cycle of the air, of the surface of consolidated sand in the open as well as in the shade of a shrub, and of the burrow of the tortoise 40 cm from the entrance (cf. also the graph).

Table 1

Temperature Records on June 2, 1938

Place \ Hours	6	8	10	12	14	16	18	20	22	1
Air $t°$ 1 m above ground level	18.3	23.5	31	37	39.8	37.7	36	30	26.2	19.2
Soil surface $t°$ (dense sands)	16.5	29	47	55	59.5	49.5	40.5	28.5	23.1	17.2
$t°$ of ground surface under shrub within 20 cm of the ground	17	25	33	42	44	41	38.2	30.5	25.1	—
$t°$ in burrow	25	27	28	30	31.5	32.2	31.0	30.0	29.5	24.8

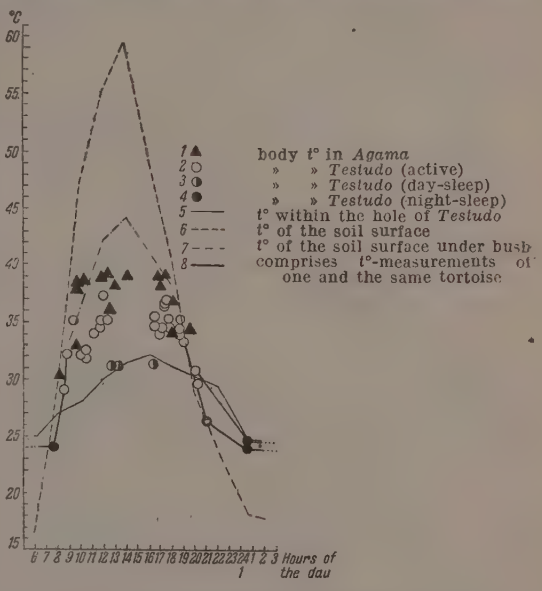
Note. A number of temperature measurements made on other days yield similar results.

A burrow, keeping fairly even temperature within a 24-hour period, provides the tortoise with an abode both from freezing at night and overheating in day time. The same graph shows changes in body temperature of an agama within a 24-hour period (night measurements have not been taken).

These, as well as round-headed lizards Radde, stopping in the morning at the exit of their burrow, spend some time on warming in the sunlight. Therefore such active (running and insect-chasing) specimens sustain a body temperature above 30° C. At sunset before retiring to the burrow, these lizards show a rather sharp drop in body temperature, the round-headed lizards Radde, for instance, down to 28—29° (their movements becoming notably

slower). During the hotter part of the day the agama escapes overheating by hiding in bushes or staying at the mouth of the burrow, and the round-headed lizard Radde, by sitting in the shade (often of tiny little shrubs) or by climbing on the top of small mounds of drifting sand. The advantage, gained by an agama through climbing up a bush, has already been described in literature(1); it may as well be seen from our table, showing that the temperature of the air 1 m above ground (39.8°) is almost 20° below the temperature of the soil (59.5°).

Even the shadow cast by a very tiny bush (under which the round-headed lizard Radde is usually taking refuge) brings down the temperature of the ground surface from 59.5 to 44.0°, i. e. causing a drop of 15.5° (see graph).



Species of sandy country: auriculate round-headed lizard (*Phrynocephalus mystaceus*), round-headed sand lizard (*Phr. interscapularis*) shun overheating in day-time—the former by climbing on the crests of barchans (crescentic sand dunes, Kashkarov) or hiding in the shade of shrubs, the latter also by sitting under bushes, and to escape the most burning heat by digging in the earth under their shadow. Besides, these two species stop in their locomotion over the ground and raise themselves upon their limbs (particularly, during the hotter hours of the day), which is also a means to get some relief from the effect of extremely high temperatures of the ground surface.

Table 2

Temperature Records on June 2, 1938

Place \ Hours	6	8	10	12	14	16	18	20	22
Air $t°$ 1 m above ground	18.3	23.5	31.0	37.0	39.8	37.1	36.0	30.0	26.2
Air $t°$ 4 cm above sand *	17.4	25	36	40	43	42	37	30	25
Air $t°$ 2 cm above sand ** . . .	17	26	39	43	46	43	39	30	24
$t°$ of the sand surface	16	32	45	57	63.2	54	44	28	22
$t°$ of the sand surface under a thick bush	18.4	23	30	36	45	41	36	29	26

Note. Clear weather all day long. Wind blowing with a velocity of 2—3 m/sec from 12 a. m. till 4 p. m.

* Height of the body of the auriculate round-headed lizard, standing up on stretched limbs.

** Do of the round-headed sand. lizard.

During the day the body temperature in all the enumerated reptiles is regulated by the behaviour of each animal chosing a place of abode with a temperature at the given time next to the optimum.

Both species of round-headed sand lizards, as well as the reticulate scapteira, spend the night dug in the sand, the auriculate round-headed lizard at a depth in the neighbourhood of 10 cm, the round-headed sand lizard excavating a small temporary hole to a depth of about 5—8 cm, and finally, the scapteira burrowing at various depths, but not any less than 10—15 cm.

Burrowing in sand even at a depth of 5 cm is sufficient to escape any too sharp fluctuation in surface temperatures. The foregoing is illustrated by the 24-hour soil temperature curve, according to the author's observations at the Sand Station of All-Union Institute of Plant Industry (Repetek, Turkmen SSR) on May 17, 1938.

Table 3

Depth \ Hours	8	10	12	14	16	18	20	22	24	2	4	6	$t°$ fluctuations
Surface	35	39	51	56	50	36	27	22	18	16	16	21	40°
5 cm	25	31	39	43	43	39	34	20	26	24	23	23	20°
10 »	26	28	32	35	37	37	36	33	31	29	27	26	11°
40 »	29.4	29.4	29.2	29.4	29.5	29.8	30	30.2	30	30	26	29.8	0.8°

And yet the body temperature of reptiles burrowing in sand for the night falls rather abruptly. Thus, individual measurements of body temperature recorded in auriculate round-headed lizards, burrowing in sand, at night and at dawn, show that their body temperature drops down to 20—23° C ($t°$ of the surrounding sand being 18.8—22°), while during the diurnal period of activity it is 34—36° C.

Of great interest is the body temperature of nocturnal forms, which are active under conditions of lower night temperatures. A few measurements taken of body temperature in geckos (*Teratoscincus scincus*) indicate that the sand surface temperature being +18—20°, these lizards maintain a body temperature from +22 to +25°. The same applies to small steppe boas (*Eryx tataricus*), crawling at night over the sand surface, which have a body temperature +22 —28°, the soil temperature at the same time being +20—25°. Both boas and geckos are quite active, in spite of the relatively low temperature. It is worth mentioning that the aforesaid gecko burrows in day time in sand only to a depth of 8—10 cm, and is apparently subjected during its day nap to heating, which raises its body temperature above the one it is sustaining during the period of activity.

In summarizing, we may state that our material, though fragmentary, corroborates in the main the above-mentioned thesis of A. V. Ryumin in relation to diurnal species. However, the body temperature of nocturnal species is evidently not subject to the rule indicated above, as they are active and move around with great speed, maintaining a temperature from about 22° to 25°, i. e. 10° less, than in diurnal forms.

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ZOOLOGY

ON THE QUESTION OF VARIATION IN STRUCTURE OF THE ALIMENTARY CANAL IN STURGEONS IN CONNEXION WITH NUTRITION

By A. BELOGUROV

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The nature of food consumed by fishes affects their general structure. In connexion with one or another kind of nutrition, not only the organs for intercepting food, but the intestine as a whole is subject to changes. The major part of research work has been dealing with adaptability to nutrition of the region of the alimentary tract between the mouth and stomach. However, investigations on the adaptation to nutrition of the stomach and intestine have been few in number. But even these few investigations have produced evidence that the adjustment to a particular kind of food involves the intestine and the stomach. We shall not go into detail regarding this question, as it has been sufficiently elucidated in various text books on fishing and ichthyology. Our task should be to gain further insight into the relationship between nutrition and volume (weight) of the intestine in fishes closely allied in the systematic position, but feeding on different kinds of food. We are fully aware of the fact that by determining merely the volume (weight) of the intestine we shall not be able to disclose all the specific features of adaptability of the intestine to nutrition, but we believe that by recording variations in the volume (weight) of intestines in fishes, systematically closely related to one another, we shall contribute to the solution of the problem. For the purpose of investigation, we have taken sturgeons (sturgeon—*Acipenser güldenstaedti* Brandt, great sturgeon—*Acipenser stellatus* Pall., and another kind of sturgeon—*Acipenser huso* L.), the feeding of which has been the object of a special study conducted by us.

On the structure of the intestine (in sturgeons) and correlation of its separate regions several works have been published, not treating, however, of structure of the intestine from the aspect of its adaptation to nutrition. The aforesaid works contain either general description of the intestine([1]), or measurements of separate parts of the enteric canal([2,3]). We have not found therein any direct answer to the question raised by us, but have taken into consideration some of the conclusions drawn by the above-mentioned authors. Both Tikhij and Behning arrive at the conclusion, that there is a definite relationship between separate regions of the enteric canal, and that such relationship is subject to some deviations. The latter circumstance has prompted us to use as extensive material as possible for purposes of our investigation.

Intestines of sturgeons have been assembled during the expedition carried out by the Scientific Research Institute of Zoology of the Moscow State University in the Caspian Sea in 1934. The material was collected in the North and South Caspian. There were assembled altogether 117 intestines of sturgeon, 72 of *A. huso* and 22 of *A. stellatus*. Qualitatively the material was not alike: as far as the two former kinds are concerned, the material comprises mostly mature fishes, and as to the latter, it is made up of young specimens. It should be noted that here was a slight difference between the weight of the young of *A. stellatus* (the largest kind among sturgeons) and that of adult specimens of *A. huso* and *A. güldenstaedti*.

The collected material was treated in the following manner: the entire intestine (without the oesophagus), after having been emptied of food, was weighed, and the weight compared with that of the body of the fish. The oesophagus had to be removed, because during assemblage of material the oesophagus was not always clearly distinguishable. Besides the intestines, the pyloric caeca were weighed, and their weight was compared to that of the intestine. Special attention was paid to the volume of pyloric appendages, for as soon as individual intestines were compared, there became evident the difference in the mass of pyloric caeca in the *A. güldenstaedti*, *A. huso* and *A. stellatus*, and furthermore, Svetovidov [4] attempted to show the correlation between nutrition and the number of pyloric appendages in 5 families of fish: *Clupeitae*, *Thymallidae*, *Pleuronectidae*, *Gadidae* and *Percidae*.

The results of our observations, after having received biometrical treatment, are summarized in the Tables 1 and 2 given below.

Table 1

Ratio of the Weight of the Intestine (without oesophagus) to the Body Weight in *A. güldenstaedti*, *A. huso* and *A. stellatus* (male and female) in %

Classes	1	2	3	4	5	6	7	8	*n*	*M*	σ	*m*
Sturgeon of North Caspian	—	3	16	28	14	7	4		72	4.8	1.2	0.14
Sturgeon of South Caspian	2	9	14	9	5	2	4		45	4.12	1.56	0.23
Sturgeon throughout the entire Caspian Sea . . .	2	12	30	37	19	9	8		117	4.5	1.45	0.13
Acipenser huso of North Caspian	—	12	17	7	5	1	—		42	3.8	1.2	0.19
Acipenser huso of South Caspian	5	21	11	3	—	—	—		40	2.8	0.78	0.12
Acipenser stellatus throughout the entire Caspian	—	11	4	3	—	—	—		18	3.05	0.75	0.18

The ratio of the difference between the means to the standard error of the difference, according to formula $\frac{M_1 - M_2}{\sqrt{m_1^2 + m_2^2}}$

1) for the sturgeon *(A. güld.)* of North and South Caspian . . . . . 2.6 difference not essential
2) for *A. huso* of North and South Caspian . . . . . 5.0 » essential
3) for *A. güldenstaedti* and *A. huso* of North Caspian . 4.3 » »
4) for *A. güldenstaedti* and *A. huso* of South Caspian . . 5.1 » »
5) for *A. güldenstaedti* and *A. stellatus* . . . . . 6.6 » »
6) for *A. huso* of North Caspian and *A. stellatus* . . . . 2.9 » not essential
7) for *A. huso* of South Caspian and *A. stellatus* . . . . 1.2 » » »

Table 2

Ratio of Weight of Pyloric Caeca to the Weight of Intestine (without oesophagus) in *A. güldenstaedti*, *A. huso* and *A. stellatus* (male and female) in %

Classes	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	*n*	*M*	σ	*m*
Sturgeon (*A. güld.*) of North Caspian	—	—	—	—	1	4	2	2	7	8	11	10	6	3	1	2	2	1		60	12.5	2.7	0.35
Sturgeon (*A. güld.*) of South Caspian	—	1	2	3	2	3	4	1	4	6	3	5	6	2	1	1	1	—		44	11.2	3.3	0.5
Acipenser huso of North Caspian	1	3	17	7	8	6	—	2	—	—	—	—	—	—	—	—	—	—		39	5.4	1.45	0.24
Acipenser huso of South Caspian	1	4	5	7	10	8	4	1	—	—	—	—	—	—	—	—	—	—		40	6.1	1.3	0.20
Acipenser stellatus throughout the entire Caspian	—	—	—	1	2	6	4	5	2	1	1	—	—	—	—	—	—	—		22	8.54	1.5	0.32

The ratio of the difference between the means to the standard error of the difference, according to formula $\frac{M_1 - M_2}{\sqrt{m_1^2 + m_2^2}}$:

1) for the sturgeon *(A. güld.)* of North and South Caspian . . . . . 2.4 difference not essential
2) for *A. huso* of North and South Caspian . . . . . 2.3 » »
3) for *A. huso* and *A. güld.* of North Caspian . . . . . 19.9 essential
4) for *A. huso* and *A. güld.* of South Caspian . . . . . 9.4 » »
5) for *A. stellatus* and *A. güld.* throughout the entire Caspian . . . . . 8.4 » »
6) for *A. stellatus* and *A. huso* of North Caspian . . . . 7.8 » »
7) for *A. stellatus* and *A. huso* of South Caspian . . . . 5.4 » »

An analysis of the material computed in Table 1 shows that the volume (weight) of the intestine (without the oesophagus) varies in *A. güldenstaedti*, *A. huso* and *A. stellatus*. In the sturgeon (*A. güldenstaedti*) captured in the North and South Caspian the ratio between the weight of the intestine and the body weight of the fish does not show any appreciable difference, the average constituting 4.5%. In *A. huso* of North and South Caspian these variations are significant: for the specimens of North Caspian the ratio is 3.8%, for those of South Caspian 2.8%, but in general the ratio of the volume (weight) of the intestine to the body weight is less than in *A. güldenstaedti*. In this relation, the *A. stellatus* occupies an intermediate position between the North and South Caspian specimens of *A. huso*, the volume (weight) of its intestine being less than that of the *A. güldenstaedti*. The material presented in Table 2 illustrates that *A. güldenstaedti* has the largest volume of pyloric caeca, namely 12.5%, *A. stellatus* following next with 8.5%, and ultimately *A. huso* with the least number of pyloric appendages, namely: North Caspian *A. huso*—5.4% and South Caspian—6.1%.

A comparison of the results obtained from our investigations and those of the ichthyologists, who had been concerned with morphometric investigations of sturgeons, may be of interest. Investigations by Belyaev [5] have shown a divergence in the majority of morphometrical characteristics: the amount of dorsal and lateral scales, the number of the dorsal fin rays and the number of the gill-rackers; disparity of plastic characters is likewise pointed out by Belyaev, but he does not discern any individual populations

or races among various kinds of sturgeon. Our observations, as stated above, have not disclosed any essential differences. According to Borzenko (6), who had studied *A. huso*, there is slight divergence in morphometrical and plastic characters between the Volga (North Caspian) and Kura (South Caspian) *A. huso*, but a sharp disparity is observed in the rate of growth (the morphometric and plastic characters being also taken into account) of these specimens, justifying his differentiation of races of *A. huso* in the Caspian. As far as *A. huso* is concerned, our investigations have pointed out essential differences, which ought to be considered when races of *A. huso* will be established.

Our investigations on nutrition of sturgeons of the Caspian Sea (as well as data contained in literary works) have emphasized that there are essential divergences in the kinds of food consumed by *A. güldenstaedti*, *A. stellatus* and *A. huso*. These fishes belong to different ecologic groups. The principal food of the *A. güldenstaedti* consists of bottom fish of the family *Gobiidae* (genera *Gobius* and *Benthophilus*), mollusks and crustaceans (*Gammaridae* and *Corophiidae*). *Chironomidae* too play an essential part in the nutrition of *A. güldenstaedti*. The *A. huso* feeds on fish (chiefly on *Gobiidae*), crustaceans and *Chironomidae*, mollusks constituting a negligible part of its nutrition. As to the great sturgeon, it is ichthyophagous and feeds both on bottom and pelagic fish. The «coarsest and hardest» kind of food is presented by mollusks, due to their shells, while that consisting of fish, *Chironomidae* and *Crustacea* is the «softer» variety. This factor, as we see it now, has also had its effect on the volume (weight) of the intestine in various kinds of sturgeon. The relatively greater volume of intestine in the *A. güldenstaedti* is evidently due to the fact that, unlike the *A. stellatus* and *A. huso*, mollusks constitute part of its nutrition.

Svetovidov's assertion, that «the bigger the organisms consumed by one or another kind of fish, the greater the number of its pyloric caeca» is not exemplified in the case of sturgeons, for though the great sturgeon (*A. stellatus*) feeds naturally on more bulky food than the common sturgeon (*A. güldenstaedti*), the volume of pyloric caeca in the former is less than in the latter.

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